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J. E. Michaels[1]

Yih-Hsing Pao
Fellow ASME

Department of Theoretical and Applied Mechanics, Cornell University, Ithaca, NY 14853

Determination of Dynamic Forces From Wave Motion Measurements

An experimental method has been developed for generating oblique forces with known orientations and time histories. Recorded signals from several forces were analyzed by an iterative deconvolution method to determine their orientations and time histories. The recovered values agree closely with the exact ones for these controlled sources. These experiments are a valuable test of source characterization methods that may be applied to seismic data from earthquake sources or to signals recorded from the acoustic emission of cracks.

1 Introduction

A time-dependent concentrated force with fixed orientation that is applied to a structure generates wave motion which may be recorded on the surface. If the location, time history, and orientation of the force are known, the displacement response can be calculated for simple structures for which the Green's functions are known. To record the response to either a known or unknown source, a transducer can be mounted on the surface of the structure. In this paper a method is described and demonstrated to experimentally determine the orientation and time history of an oblique force applied to the surface of an elastic plate.

A closely related problem is determining the time history of a concentrated force of known orientation. Goodier et al. [1959] solved an integral equation to calculate the time history of a vertical force applied to a half-space from the far-field response. Hsu et al. [1977] and Michaels et al. [1981] discretized and inverted a time convolution integral to determine the time history of a vertical force applied to a plate from the near-field response. An important factor in the success of their work was the availability of an artificial source, which is generated by fracturing a glass capillary against the structure surface. As was first noted by Breckenridge et al. [1967], this source is a concentrated vertical force that has the time-dependence of a step-like unloading function.

The response of a structure to a force of known orientation is given by a convolution in time of a source function with a single Green's function. Thus, the problem of determining the source function from the response may be solved by deconvolution. However, for a force of unknown orientation, the response is given by a convolution of the source time function with a linear combination of Green's functions, where the unknown coefficient of each Green's function is proportional to a direction cosine of the force. Therefore, methods for deconvolution with a single Green's function cannot be directly applied to determine both the source time function and the direction cosines. In this paper, a deconvolution method recently developed by Michaels and Pao [1985] for multiple Green's functions is applied to experimental data.

2 Theory

Before the orientation and time history of an oblique force can be determined from the measured wave motion, it must be understood how the response depends upon the source, the medium, and the receiver. It is assumed here that the medium is an infinite elastic plate and that the receivers are piezoelectric transducers sensitive to normal motion.

Displacement Response in a Plate. Consider the plate geometry shown in Fig. 1. In cylindrical coordinates (r, θ, z), the source is located at $\mathbf{x}^o = (0, 0, 0)$ and a typical receiver is at $\mathbf{x} = (r, \theta, h)$, where h is the plate thickness. It is assumed that the horizontal dimensions of the plate are large enough so that it can be modeled as infinite in extent.

The Green's displacement tensor, $G_{ij}(\mathbf{x}, t; \mathbf{x}^o)$, is defined to be the displacement response in the ith direction at $\mathbf{x}$ and t due to an impulsive concentrated force of unit magnitude in the jth direction at $\mathbf{x}^o$ and $t=0$. Thus, for a point force $F_j(\mathbf{x}^o, t)$ acting at $\mathbf{x}^o$ that is zero for $t<0$, the resulting displacement is,

$$\begin{aligned} u_i(\mathbf{x}, t) &= \sum_{j=1}^{3} \int_0^t d\tau G_{ij}(\mathbf{x}, t-\tau; \mathbf{x}^o) F_j(\mathbf{x}^o, \tau) \\ &= \sum_{j=1}^{3} G_{ij}(\mathbf{x}, t; \mathbf{x}^o) * F_j(\mathbf{x}^o, t) \end{aligned} \tag{1}$$

In this and subsequent equations, an asterisk denotes a convolution integral in the time variable.

It is assumed here that the orientation of the oblique force F_j does not change with time. It then may be expressed as,

$$F_j(\mathbf{x}^o, t) = f_j(\mathbf{x}^o) s(t) \tag{2}$$

The function $s(t)$ is the time history of the force, which is the same for all three components of F_j. The vector $f_j(\mathbf{x})$ is the time-independent orientation. This decomposition in equation (2) is not unique because it is defined only to within a scale factor.

[1] Present address: JTM Systems and Consulting, Inc., Terrace Hill, Ithaca, NY 14850.

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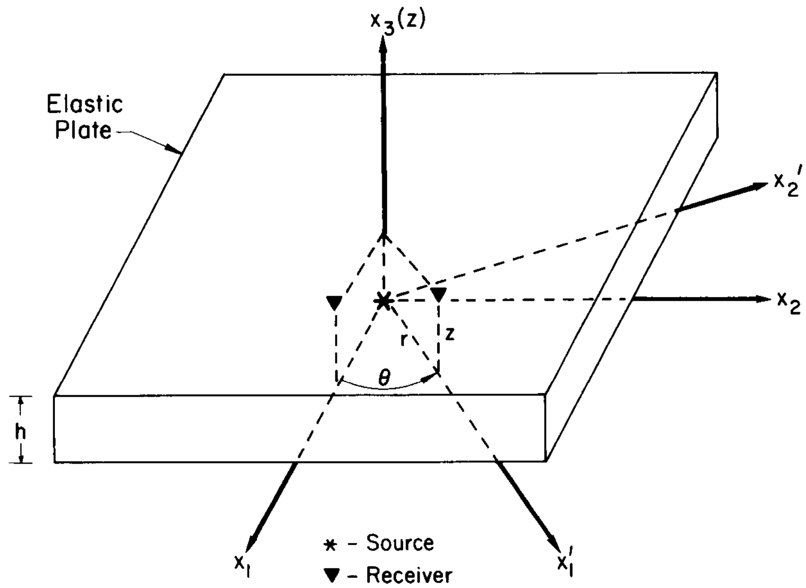

Fig. 1 Source and receiver geometry for an infinite elastic plate

Since the plate is axially symmetric and the source is located on the x_3 (or z) axis, the normal displacement $u_3(r, \theta, h, t)$ may be evaluated from the Green's functions at $\mathbf{x}=(r, 0, h)$ by rotating the components of the force f_j,

$$u_3(r, \theta, h, t) = \left\{ \sum_{j=1}^{3} G_{3j}(r, 0, h, t; \mathbf{0}) f_j{}' \right\} * s(t) \qquad (3)$$

The components $f_j{}'$ are,

$$\begin{aligned} f_1{}' &= f_1 \cos\theta + f_2 \sin\theta \\ f_2{}' &= -f_1 \sin\theta + f_2 \cos\theta \\ f_3{}' &= f_3 \end{aligned} \qquad (4)$$

Equation (3) may be simplified by noting that $G_{32}(r, 0, h, t; \mathbf{0})$ vanishes identically because of the axial symmetry of the plate (Ceranoglu and Pao, 1981).

$$\begin{aligned} u_3(r, \theta, h, t) = \{ & (f_1 \cos\theta + f_2 \sin\theta) G_{31}(r, 0, h, t; \mathbf{0}) \\ & + f_3 G_{33}(r, 0, h, t; \mathbf{0}) \} * s(t) \end{aligned} \qquad (5)$$

Thus, if the Green's functions are known, the normal displacement response to a known oblique force may be calculated.

Deconvolution with Multiple Green's Functions. To determine f_1, f_2, f_3 and $s(t)$ from the measured normal displacement u_3, we must consider the problem of deconvolution where the kernel is a sum of Green's functions with unknown coefficients. The method used is described in detail by Michaels [1984] and Michaels and Pao [1985] and is only briefly reviewed here.

Equation (5) for displacement is of the form,

$$u(t) = \left\{ \sum_{m=1}^{M} c_m G_m(t) \right\} * s(t) \qquad (6)$$

where we identify,

$$\begin{aligned} u(t) &= u_3(r, \theta, h, t) \\ M &= 2 \\ c_1 &= f_1 \cos\theta + f_2 \sin\theta \\ G_1(t) &= G_{31}(r, 0, h, t; \mathbf{0}) \\ c_2 &= f_3 \\ G_2(t) &= G_{33}(r, 0, h, t; \mathbf{0}) \end{aligned} \qquad (7)$$

In general, there are several receivers, and equation (6) is valid for each receiver but with different c_m and $G_m(t)$.

The first step is to calculate c_1, c_2 and $s(t)$ at each receiver location by an iterative deconvolution procedure. The coefficients c_1 and c_2 are first set to non-zero initial values. Using these values, the source time function $s(t)$ is estimated by least squares deconvolution. Then, improved estimates of c_1 and c_2 are calculated from the estimate of $s(t)$, again by least squares. This procedure of alternately calculating c_m and $s(t)$ is continued until they converge to stable values.

The final estimate of $s(t)$ is obtained by averaging the signals obtained by deconvolution at all of the receivers. Similarly, f_3 is obtained by averaging the coefficient c_2. However, the iterative deconvolution procedure does not recover f_1 and f_2 directly. They are imbedded in the coefficient c_1 as shown in equation (7). Thus, to calculate f_1 and f_2, there must be at least two receivers located at different angular positions. For more receivers, a least squares minimization is performed to obtain f_1 and f_2 from the coefficients c_1 at all of the receivers.

Thus, the iterative deconvolution method yields $\hat{\mathbf{f}}$ and $\hat{s}(t)$, estiamtes of $\mathbf{f}$ and $s(t)$, the parameters of the oblique force. As discussed previously, $\hat{\mathbf{f}}$ and $\hat{s}(t)$ may be mulitiplied and divided, respectively, by an arbitrary scale factor. Here we use the convention that $\hat{\mathbf{f}}$ is a unit vector, and thus scale $\hat{s}(t)$ such that

$$\sum_{i=1}^{3} \hat{f}_i^2 = 1 \qquad (8)$$

Since $\mathbf{f}$ and $\hat{\mathbf{f}}$ are both unit vectors, the angle between them is given by,

$$\Delta\phi = \cos^{-1}(\mathbf{f} \cdot \hat{\mathbf{f}})$$

This angle is a measure of the error in determining the orientation of the oblique force.

Transducer Characterization. The piezoelectric transducers used in the work reported here are primarily sensitive to normal velocity. They are also small in size such that a point receiver model is appropriate. Thus, we assume that the output of the amplifier $a(t)$ can be expressed as,

$$a(t) = T(t) * \frac{du_3(\mathbf{x}, t)}{dt} = T(t) * \dot{u}_3(t) \qquad (10)$$

In this equation, $T(t)$ is the transfer function for the transducer. It characterizes not only the transducer but the coupling of the transducer to the structure and the recording

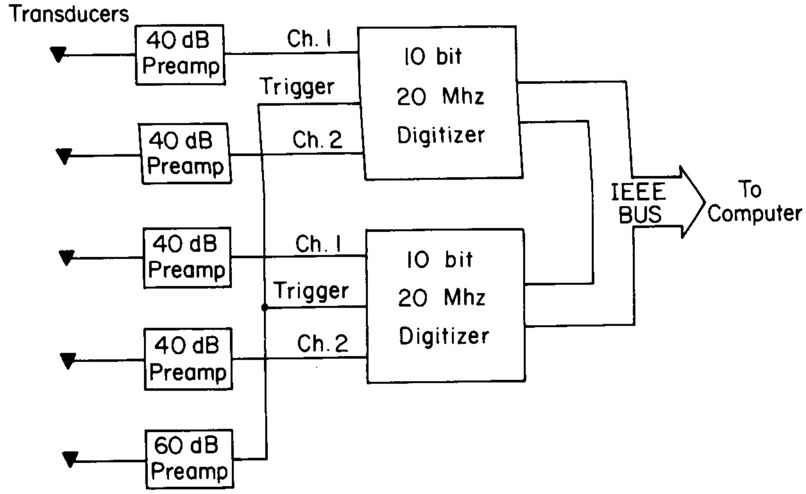

Fig. 2 Data acquisition equipment for oblique force measurements

equipment (cabling, amplifiers, transient recorders). It is further assumed that $T(t)$ is of finite duration.

The first step in the characterization procedure is to determine the transfer function $T(t)$ for each transducer. This is done by measuring the response to the fracture of a glass capillary. This dynamic source is initiated by slowly pressing vertically against a capillary tube until it fractures. As shown by Breckenridge et al. [1967], this source is a concentrated vertical force that has the time dependency of a step-like unloading function. If the finite rise time is neglected, the time dependence of the source is represented by a step function $H(t)$, and we have,

$$\begin{aligned} \mathbf{f} &= -\mathbf{e}_3 \\ s(t) &= H(t) \end{aligned} \tag{11}$$

The resulting normal velocity at $\mathbf{x} = (r, \theta, h)$ is,

$$\begin{aligned} v_c(t) &= -\frac{d}{dt}\{G_{33}(\mathbf{x}, t; \mathbf{0}) * H(t)\} \\ &= -G_{33}(\mathbf{x}, t; \mathbf{0}) \end{aligned} \tag{12}$$

If $a_c(t)$ denotes the signal generated by breaking the capillary, equation (10) then yields

$$a_c(t) = T(t) * v_c(t) \tag{13}$$

Since $a_c(t)$ is measured and $v_c(t)$ is known from calculation of the Green's function G_{33}, the transfer function $T(t)$ can be evaluated by deconvolution.

Now let $a(t)$ be the measured signal from the unknown source. From this signal, the iterative deconvolution procedure recovers a source time function $S(t)$. This $S(t)$ is the convolution of $\dot{s}(t)$, the derivative of the source time function, with the transducer transfer function $T(t)$.

$$S(t) = T(t) * \dot{s}(t) \tag{14}$$

Since both $S(t)$ and $T(t)$ are known, $\dot{s}(t)$ can be calculated by deconvolution. It must be numerically integrated to obtain the source time function $s(t)$. Therefore, the source time function can be recovered using a transducer that is not a displacement sensor, but that is sensitive to vertical velocity.

If the transfer function is band-limited, those frequency components of $s(t)$ not present in $T(t)$ cannot be recovered by deconvolution. For example, if the transducer is not sensitive to high frequencies, the recovered $s(t)$ will be missing high frequency information and fast rise times cannot be accurately recovered.

3 Experimental Methods

Experiments were performed by generating oblique forces with step-like time functions on the surface of a glass plate. Signals were recorded and analyzed to determine the orientation and time dependence of several forces.

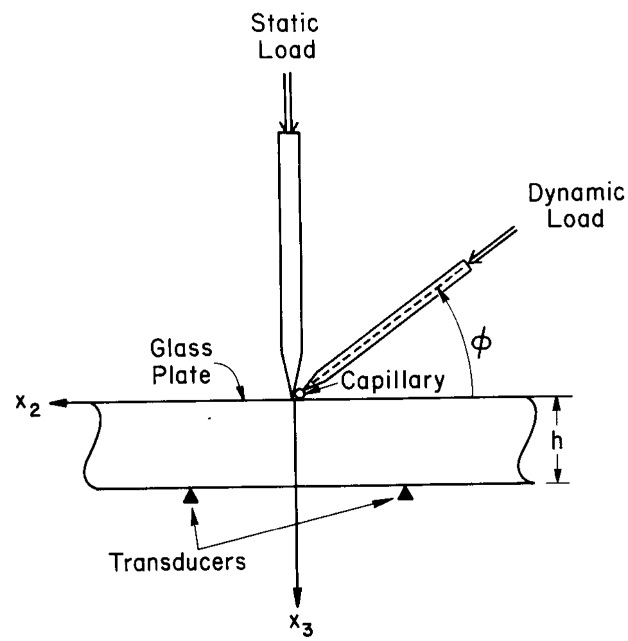

Fig. 3 Experimental setup for generating oblique forces

Specimens and Equipment. A single glass plate was used for all experimental measurements. The plate was approximately 150 mm × 150 mm in extent, and was 18.46 mm thick ($h = 18.46$ mm). The longitudinal and shear wave speeds were measured with a pulse-overlap technique, and were 5.81 mm/μs and 3.46 mm/μs, respectively.

The transducers used to record the wave motion in the plate contained circular piezoelectric crystals 1.35 mm in diameter. The frequency response of these transducers had an upper limit of approximately 1 Mhz. Since the high frequency response of the transducers was negligible, rise times faster than about 1 μs could not be accurately recovered.

The transducer voltage signals were amplified with a gain of 40 dB and then digitized and stored. An additional transducer was used to trigger the digitizers for each channel, as shown in the equipment diagram in Fig. 2. The sampling frequency was 20 Mhz ($\Delta t = 0.05$ μs), and the data were digitized with a resolution of 10 bits. Each recorded signal consisted of 201 points, which corresponds to a time window of 10 μs. This was the longest possible time window that could be recorded with no reflections from the edge of the plate.

Table I Coordinates of transducers for measurements at different radii

Transducer Number	r	θ	z
1	1.03h	0°	1h
2	2.31h	117°	1h
3	3.10h	180°	1h
4	2.06h	256°	1h

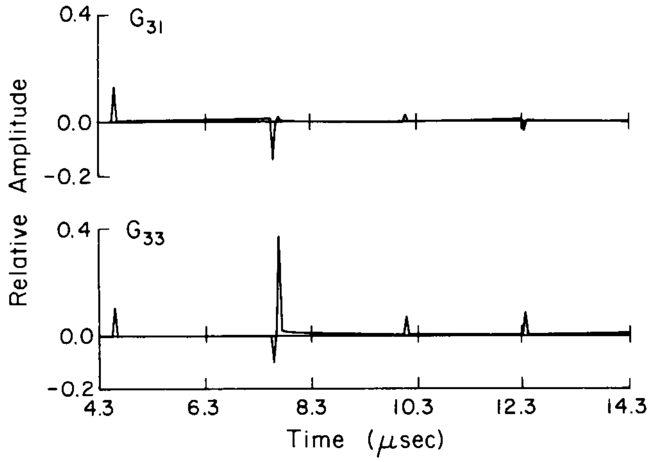

Fig. 4(*a*) Green's functions G_{31} and G_{33} at $x(r, \theta, z)$ = (1.03 *h*, 0, *h*) for a source at x^o = (0, 0, 0)

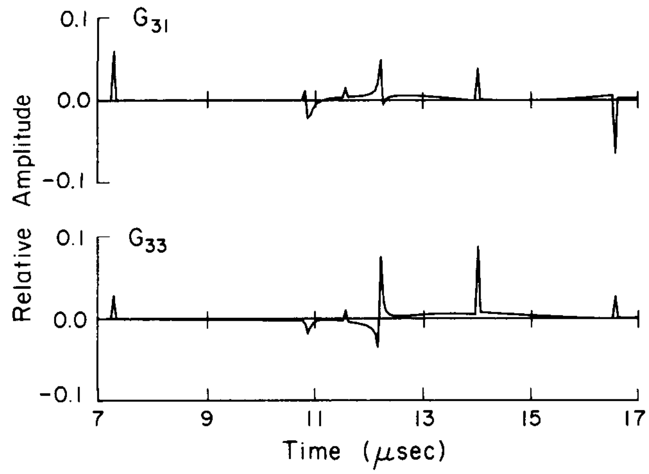

Fig. 4(*b*) Green's functions G_{31} and G_{33} at $x(r, \theta, z)$ = (2.31 *h*, 0, *h*) for a source at x^o = (0, 0, 0)

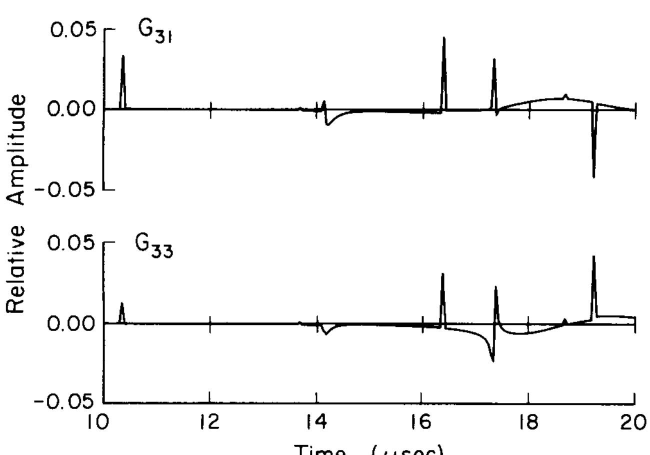

Fig. 4(*c*) Green's functions G_{31} and G_{33} at $x(r, \theta, z)$ = (3.10 *h*, 0, *h*) for a source at x^o = (0, 0, 0)

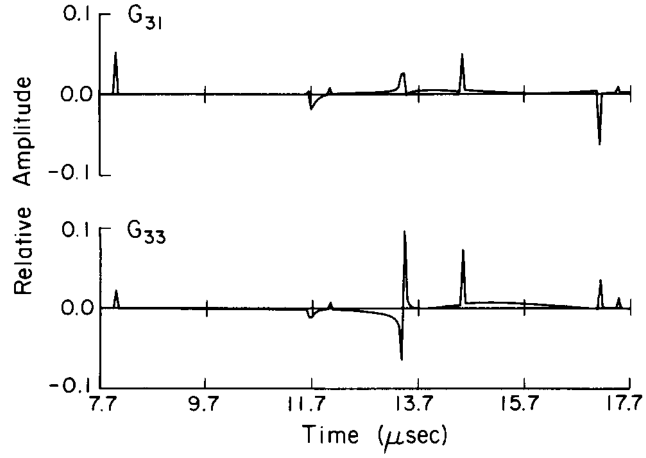

Fig. 4(*d*) Green's functions G_{31} and G_{33} at $x(r, \theta, z)$ = (2.06 *h*, 0, *h*) for a source at x^o = (0, 0, 0)

Oblique Force Generation. The experimental setup for generating oblique forces on the top surface of the plate is shown in Fig. 3. Note that the transducers are located on the bottom surface of the glass plate. The vertical rod, which was 3.18 mm in diameter and had a taper angle at the tip of about 33 degrees, was statically located onto the top plate surface. A glass capillary tube with 0.08 mm o.d. and 0.05 mm i.d. was placed in the corner between the rod tip and the plate along the x_1 axis. A second rod, which was 2.38 mm in diameter and had a taper angle of about 21 degrees, was held at an angle ϕ such that its tip was in contact with the capillary. This rod was slowly loaded along its axis until the capillary fractured. Therefore, the time dependence of this source is a step-like function, and the orientation of the force is along the axis of the second rod but in the opposite direction of the applied force, since the fracture of the capillary unloads the plate.

$$\begin{aligned} \mathbf{f} &= -\cos\phi\mathbf{e}_2 - \sin\phi\mathbf{e}_3 \\ s(t) &= H(t) \end{aligned} \tag{15}$$

Since the capillary is physically very small, it is quite difficult to position the tip of the second rod exactly on the apex of the capillary. Therefore, the measured angle ϕ may differ from the exact angle of the oblique force by as much as 5 to 10 degrees.

4 Results

Experiments were performed with four transducers that were located at different radial and angular locations. The coordinates of the transducers are listed in Table I. Since the radii are different for each transducer, the Green's functions G_{31} and G_{33} are also different. These Green's functions are shown in Figures 4(*a*)-(*d*). The ordinate is relative displacement, and is consistent for the four functions.

The Green's functions were calculated by numerically differentiating the displacement responses to forces with the time dependence of a Heaviside unit step function. Therefore, G_{31} and G_{33} are not truly the impulse responses, but the responses to a rectangular pulse of unit amplitude and width $\Delta t = 0.05$ μs. Since the width of the pulse is the same as the sampling interval, G_{31} and G_{33} may be treated as though they were impulse responses.

The step function responses were calculated by a computer program developed at Cornell University by R. Gajewski and A. Ceranoglu. The program is based upon the generalized ray theory (Pao and Gajewski, 1977), which obtains the transient displacement signals for a given time in the form of a finite series of integrals. Each integral corresponds to a particular ray path in the plate, and is evaluated numerically by Cagniard's method. The displacement signals obtained by this program are very accurate, and have been experimentally

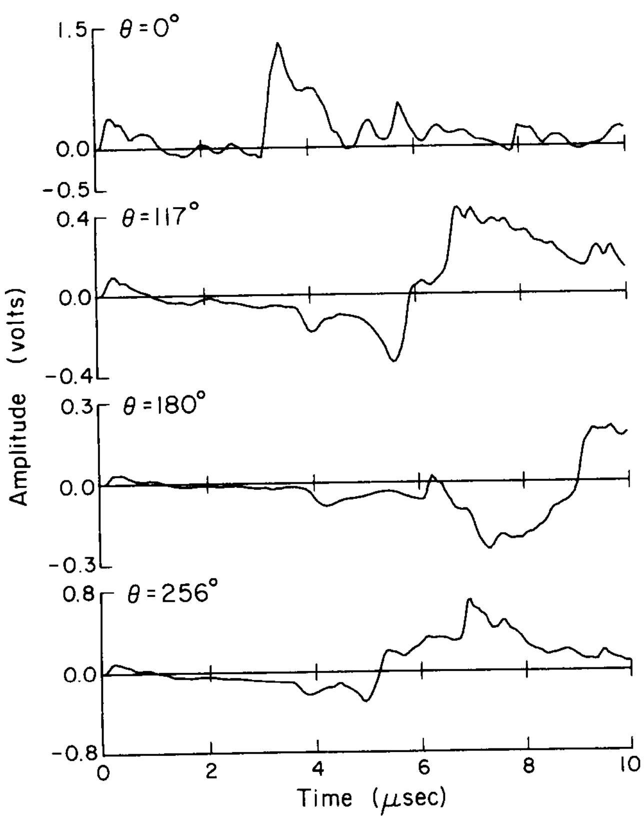

Fig. 5 Recorded waveforms from calibration source

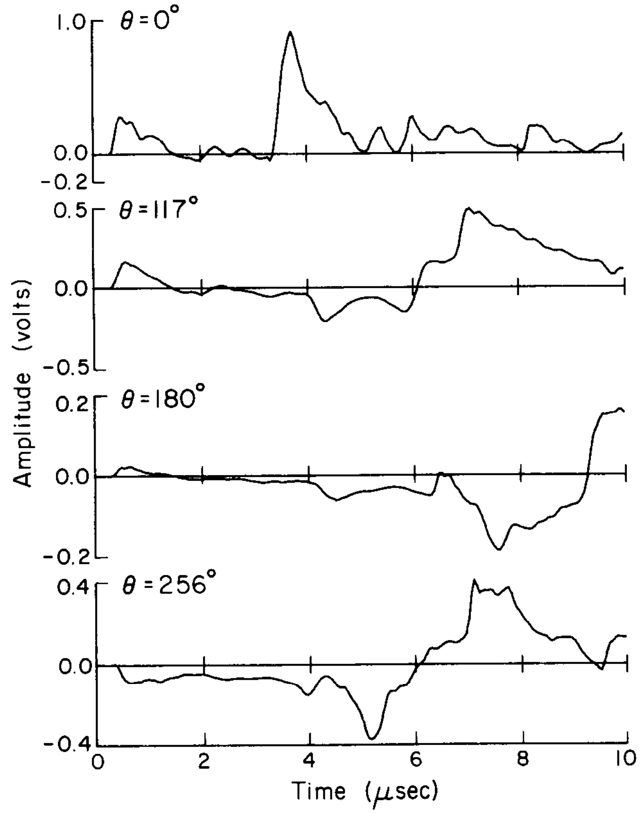

Fig. 7 Recorded waveforms for oblique force at 55 deg

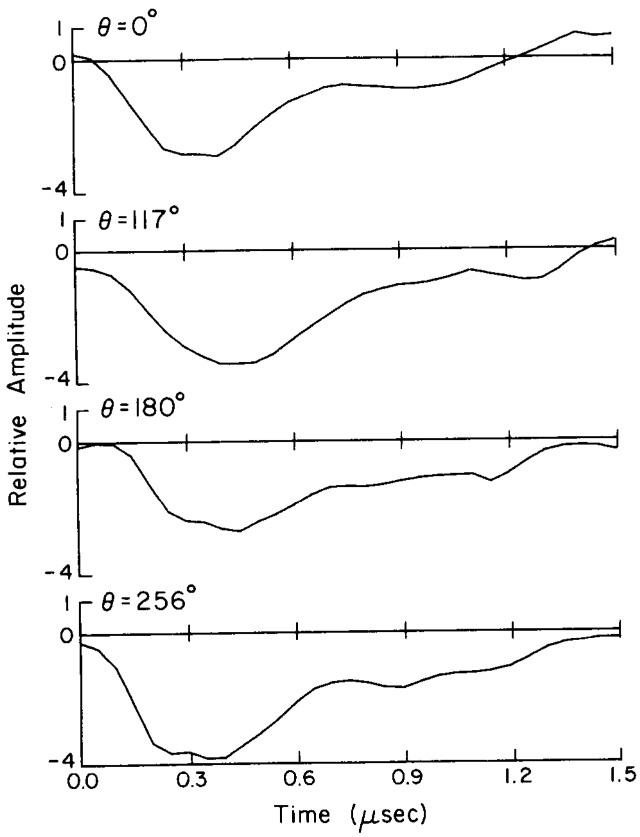

Fig. 6 Transducer transfer functions

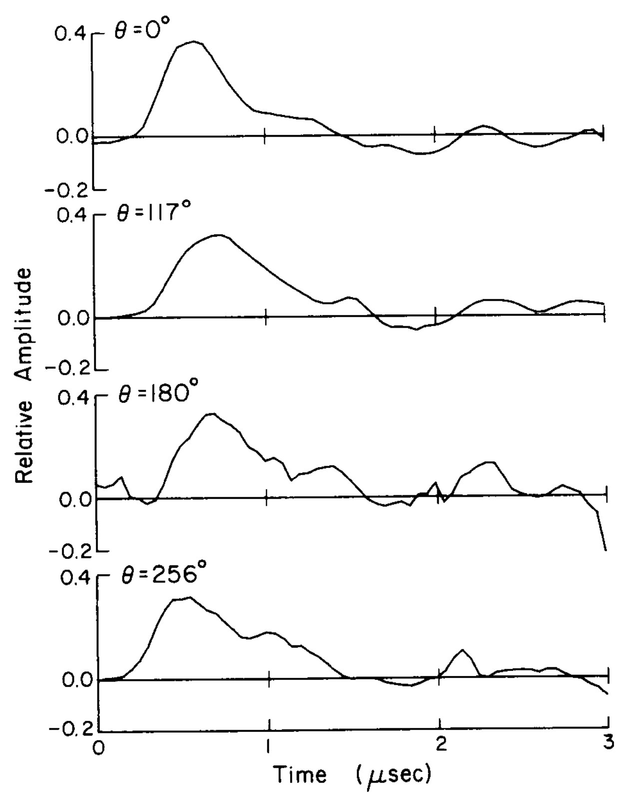

Fig. 8 Convolved functions $S(t) = \dot{s}(t) * T(t)$ recovered from 55 deg oblique force data

verified for vertical forces (Sachse and Ceranoglu, 1979, and Procter et al., 1983).

The four transducers were characterized by velocity transfer functions according to equation (10). A glass capillary 0.08 mm o.d. and 0.05 mm i.d. was broken at $\mathbf{x}^o = (0,0,0)$, and the resulting voltage signals for each transducer were digitized and stored. The average of three signals for each transducer is shown in Fig. 5. These signals were deconvolved to obtain transfer functions for each transducer, which are shown in Fig. 6. These functions are negative in sign because of an inverting amplifier.

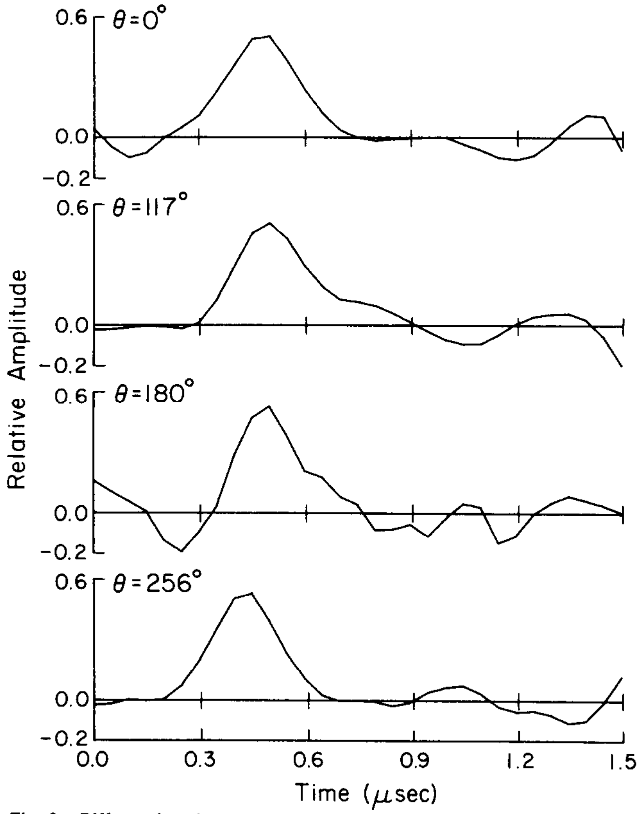

Fig. 9 Differentiated source time functions $\dot{s}(t)$ recovered from 55 deg oblique force data

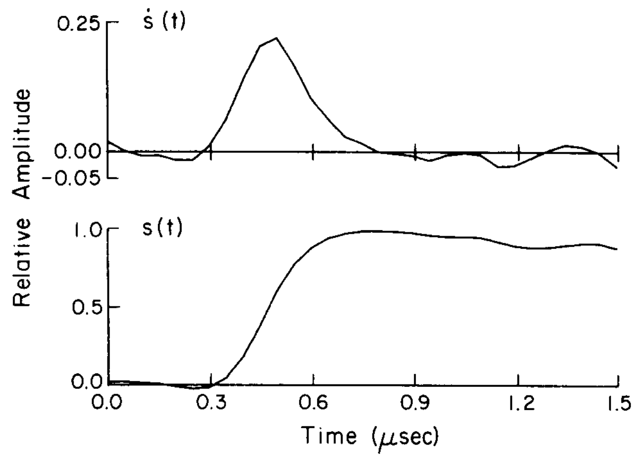

Fig. 10 Averaged function $\dot{s}(t)$ and source time function $s(t)$ recovered from 55 deg oblique force data

Oblique Force at 55 Deg. The first oblique force was generated at a nominal angle of $\phi = 55$ deg. This corresponds to an orientation of,

$$\mathbf{f} = (0, -0.5736, 0.8192) \tag{16}$$

The measured signals from each of the transducers are shown in Fig. 7.

These signals were analyzed according to the iterative deconvolution procedure described in Section 2. The recovered time functions are the convolution of the differentiated source time function $\dot{s}(t)$ with the transfer functions $T(t)$, and are shown in Fig. 8. The functions $\dot{s}(t)$ at the four transducer locations were determined by deconvolution, and are shown in Fig. 9. The final estimate of $\dot{s}(t)$ was found by first averaging the signals at the four locations, and then rescaling to correct for the normalization of $\mathbf{f}$. It is shown in Fig. 10 along with $s(t)$, which was obtained by numerical integration. The recovered $s(t)$ is a step-like function with a rise time of approximately 0.3 μs. The actual rise time could be less because of the limited high frequency response of the transducers. The amplitude scale is not absolute but is relative to the magnitude of the calibration source.

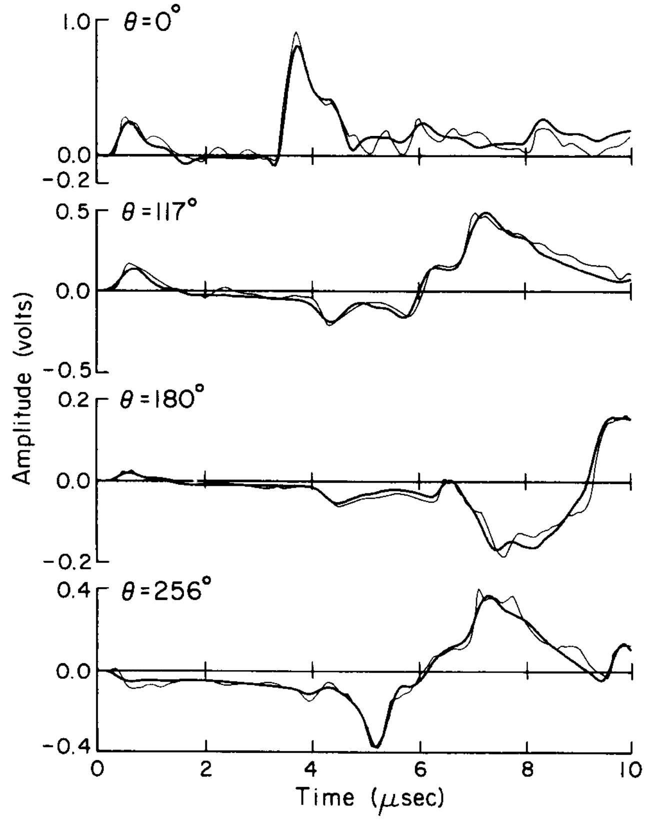

Fig. 11 Fitted transducer signals for 55 deg oblique force data

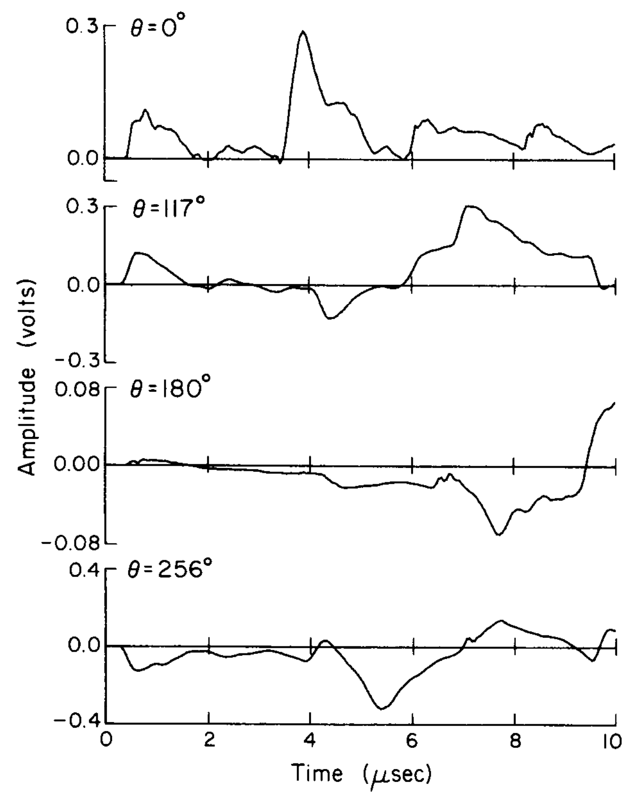

Fig. 12 Recorded waveforms for oblique force at 32 deg

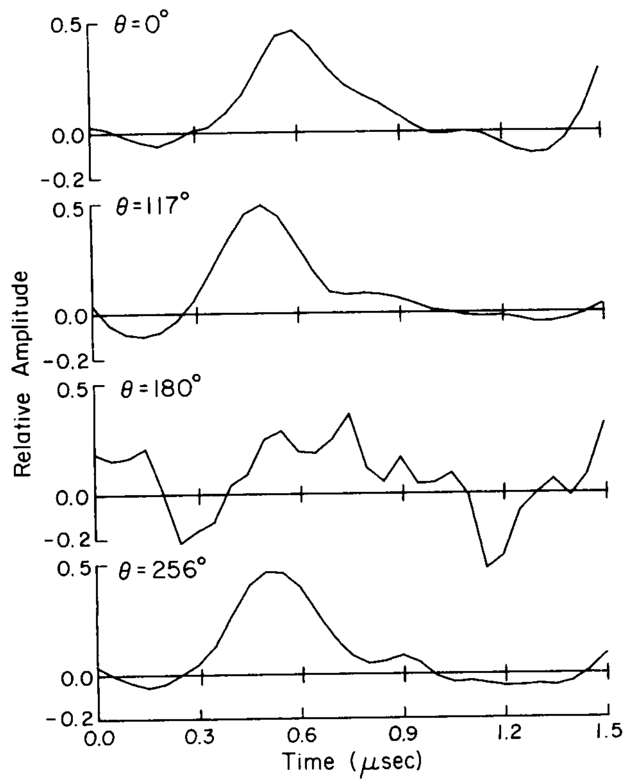

Fig. 13 Differentiated source time functions $\dot{s}(t)$ recovered from 32 deg oblique force data

The recovered orientation vector $\hat{\mathbf{f}}$ after normalization to a unit vector is,

$$\hat{\mathbf{f}} = (-0.0501, -0.5658, -0.8230) \tag{17}$$

The angular error between **f** and $\hat{\mathbf{f}}$, as calculated by equation (9), is 2.87 deg.

To evaluate how well the recovered $\hat{\mathbf{f}}$ and $s(t)$ model the data, they were used to calculate fitted transducer signals by convolution according to equations (5) and (10). These calculated signals are shown in Fig. 11. They may be compared to the measured signals in Fig. 7, which are shown as light lines in Fig. 11 for comparison. The calculated signals closely match the measured data except for some high frequency information that is not modeled by the recovered parameters.

Oblique Force at 32 Deg. The second oblique force was generated at an angle of $\phi = 32$ deg such that,

$$\mathbf{f} = (0, -0.8480, -0.5299) \tag{18}$$

The measured signals are shown in Fig. 12. Note that the signal at 180 deg is quite small in amplitude and somewhat noisy compared to the other three signals. This is because the transducer for this signal is located at a null in the radiation field for the horizontal component, and it is also at the largest distance from the source ($r = 3.10\ h$).

These signals were analyzed to obtain $\dot{s}(t)$ at each transducer, with results shown in Fig. 13. These $\dot{s}(t)$ are all quite similar except for the one obtained at 180 deg, which is very noisy and bears little resemblance to the others. This is because the measured signal at 180 deg is small in amplitude and has a lower signal-to-noise ratio than the other three. Thus, the final estimate of $\dot{s}(t)$ is the average of the signals from the first, second and fourth transducers only, and is shown in Fig. 14. Also shown in Fig. 14 is $s(t)$, which was obtained by numerical integration. It is a step-like function with a rise time of about 0.5 μs, and is very similar to the source time function obtained for the 55 deg force.

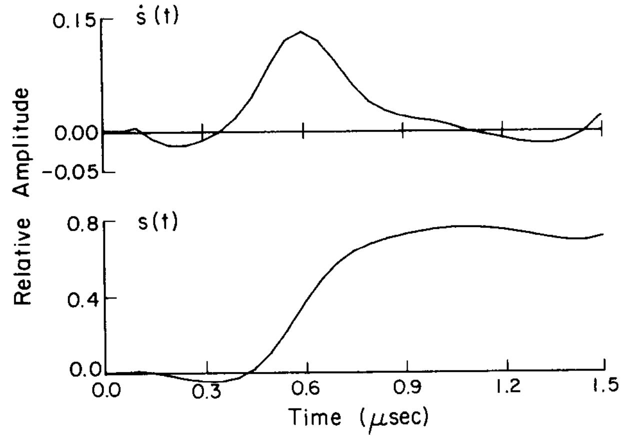

Fig. 14 Averaged function $\dot{s}(t)$ and source time function $s(t)$ recovered from 32 deg oblique force data

The recovered orientation vector $\hat{\mathbf{f}}$ after normalization to a unit vector is,

$$\hat{\mathbf{f}} = (-0.0555, -0.8923, 0.4480) \tag{19}$$

The angular error between **f** and $\hat{f}$ is 6.25 deg.

5 Summary and Conclusions

In this paper we have presented results that experimentally confirm an inverse method for determining the orientation and time history of dynamic oblique forces. In previous work, time histories of forces with known orientation have been determined by deconvolution techniques. Here, we have solved the problem of simultaneously determining the orientation as well as the time history of an oblique force. The required data are signals recorded at a minimum of two receivers that are sensitive to normal motion.

An important part of the successful demonstration of this inverse method was the development of an experimental procedure to generate controlled oblique forces. The procedure consists of fracturing a glass capillary with a load slowly applied at a known angle to the specimen surface. The resulting dynamic unloading force has a step-like time function and controlled orientation.

The key step in the inversion method is the determination of a source time function $s(t)$ and coefficients c_m of a linear combination of Green's functions, as represented by equation (6). This procedure is not limited to the characterization of oblique forces, but can be applied to many dynamic sources that are separable in time and space. The particular problem that motivated this study is that of characterizing cracks or earthquakes from recorded transient signals. The parameters to be determined for these sources are the time history and moment tensor components of the crack or earthquake. Another problem to which this method can be applied is that of recovering the spatial distribution of a separable extended source of known orientation, as was done by Chung and Sachse [1985] for synthetic data.

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