A MODELING APPROACH TO INTERPRET THE DISPERSION RELATIONSHIP FOR PIEZOELECTRIC SOURCES

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ABSTRACT. Guided wave techniques have great potential for the structural health monitoring of plate-like components. Previous research has demonstrated the effectiveness of combining laser-ultrasonic techniques with time-frequency representations to experimentally develop the dispersion relationship of a plate; the high fidelity, broad bandwidth and point-like nature of laser ultrasonics are critical for the success of these results. Unfortunately, laser ultrasonic techniques are time and cost intensive, and are impractical for many in-service applications. This research develops a complementary digital signal processing methodology that uses mounted piezoelectric elements instead of optical devices. This study first characterizes spatial and temporal effects of oil coupled and glued piezoelectric sources, and then develops a procedure to interpret and model (in the forward sense) the distortion caused by their limited bandwidth and finite size. These considerations give the background for further studies to develop the dispersion relationship of a plate having the fidelity and bandwidth similar to results provided with laser ultrasonics, but made using mounted piezoelectric sources.

INTRODUCTION

Previous research results (e.g., Benz et al. [1]) have shown that it is possible to obtain accurate dispersion curves for plates using a laser source for the wave excitation. The point-like nature and broad bandwidth of the laser source means that the individual modes in the dispersion curves are well separated and are clearly distinguishable, especially for a frequency band from 500 kHz through 5 MHz for the used excitation - and measurement system (Figure 1.). If the dispersion curves can be accurately measured, then there is quantitative information about the behavior of the individual modes. Each mode interacts with the specimen and especially its defects, so that one can detect and even characterize certain material properties and defects using these dispersion curves. When examining a similar measurement - one made with either a piezoelectric source (referred to as a “piezo disc”) or a commercial transducer (referred to as a “commercial transducer”), instead of a laser source, one recognizes a tremendous loss in information. The dispersion
curves developed using a piezoelectric source (or a transducer) are blurred (some modes are not even excited), making it difficult to distinguish between different modes, and complicating any conclusions about structural health. The motivation of this research is to develop a methodology to describe the effects of transducer and piezo disc sources (when compared to the laser source), then extract the effect of these transducer and piezo disc sources, and finally compensate for the negative influence of these sources; the ultimate goal is to obtain dispersion curves close to those developed with an “ideal” laser signal. Comparing the practicability, costs and sensitivity of transducers or piezo discs with laser sources, this methodology would have a positive impact in structural health monitoring.

EXPERIMENTS PERFORMED

The following sets of measurements are performed on an aluminum plate (Aluminum 3003 – 305 mm x 610 mm x 0.99 mm):

- One set of 61 equidistant measurements (with equal step of 0.1 mm separating each measurement location) which are performed with a laser source and a laser interferometric receiver (see Hurlebaus [2]) on the plate. Both, source and receiver are aligned such that the measurement and excitation points are in the middle of the plate, and on the same side of the plate. The largest propagation distance between source and receiver is 49 mm and the shortest propagation distance is 43 mm; this data set represents a set of symmetric propagation distances centered about a propagation distance of 46 mm. This data set of measurements will be referred to as “laser source Lamb.”

- One set of measurements on the plate performed with a commercial transducer (Panametrics V544) source and the laser interferometer receiver [2] (source and receiver on the same side of the plate). For this transducer, food oil was used as couplant. The commercial transducer source (diameter of 6 mm) is placed on the plate surface such that its center is located 46 mm away from the (point) laser receiver. This measurement will be referred to as “transducer source Lamb.”
• One set of measurements on the plate performed with a glued piezo disc source (where silver epoxy was used as glue) and the same laser interferometric receiver (source and receiver again on the same side of the plate). The piezo disc (diameter of 5.2 mm) is placed on the plate surface such that its center is located 46 mm away from the (point) laser receiver. This measurement will be referred to as “piezo source Lamb.”

**MODEL**

A scheme of the source – receiver location is depicted in Figure 2. It shows the location of a source distributed over a region of the surface $S$ with a spatial extent described by vector $r'$ and the location of the receiver described by vector $r$.

The continuous representation of the equation which is subject to the further considerations can be written as follows:

$$x(r,t) = s(t) * \int w(r') G(r,r',t) dS$$  \hspace{1cm} (1)

where $w(r')$ represents the weight for the Green’s function obtained at $r$ (excited at $r'$). The effects of the source are captured in the source time function $s(t)$. In this research the set of laser source signals $x_{LSR}(n)$ corresponds to the single Green’s functions (at different locations $j$) and the measured piezoelectric disc - or transducer signal $x_{TR}(n)$ is described by $x$. Thus, the continuous representation in Eq. (1) can be rewritten to the discrete representation in Eq. (2).

$$x_{TR}(n) = s(n) * \sum_j w_j x_{LSR_j}(n)$$  \hspace{1cm} (2)

The following observations consider the two most intuitive physical influences of a piezoelectric disc or a transducer on signal quality in the dispersion curves: temporal effects and spatial effects; that is, the influence of $s(n)$ and $w_j$ are subject to the following discussions.

**FIGURE 2.** Scheme of source – receiver location.
TEMPORAL EFFECTS

Consider the temporal effects using a synthetic source function $s(n)$ (Figure 3) only that models the possible temporal influence with exponentially damped oscillations of the transducer or the piezoelectric disc.

This synthetic source function is convolved with a single laser source Lamb measurement – only one $w$ in Eq. (2) – to obtain the contour plot of the short time Fourier transform (STFT) depicted in Figure 4. This figure confirms that a simple oscillation of the source has simply filtered the time-frequency laser representation of the signal in Figure 1. This makes physical sense by noting that a convolution of a weighted sine function with an arbitrary signal in the time domain is simply a filtering operation.

SPATIAL EFFECTS

Now, determine how “well” the piezoelectric disc and commercial transducer sources can be modeled by applying certain weight distributions to a summation of single laser measurements. Note that all the results presented in this section are based upon the plate data. Since the major interest of this research is in the effects of the source on the dispersion curves, only the first arrivals of the Lamb waves are shown, i.e., the time axis is truncated to cut off all edge reflections. Three different axially symmetric weight distributions over the circular source area are considered: a piston distribution, a Gaussian shaped distribution and an inverted Gaussian distribution.

The resulting STFTs of the three modeled laser signals will then be compared with the STFTs of the measured commercial transducer and piezoelectric disc data.

The piezo disc and transducer are modeled according to the general schematic shown in Figure 5. The effective surface of the piezoelectric disc or transducer is divided into ten rings of equal area, where each of these rings is further divided into 64 curved elements of equal area. Each elements’ area is proportional to the weight of the corresponding “laser source Lamb” signal (weighted Green’s function). This signal is found by calculating the propagation distance from the center of the elements and assigning it to the closest available distance in the measured laser set. The weights are further modified as needed to generate the desired profile (i.e., piston, Gaussian, or inverted Gaussian).

FIGURE 3. Synthetic source time function. FIGURE 4. STFT of convolution result.
FIGURE 5. Simplified scheme of how single laser source signals are assigned to the segments on the effective transducer or piezoelectric disc surface.

All STFTs for these synthetic signals are shown in Figures 6, 7, and 9 along with the actual piezoelectric disc and commercial transducer STFTs in Figures 8 and 10. Comparing Figures 6 and 7, it can be seen that the piston and the Gaussian profiles result in similar dispersion curves. They both have a “hot spot” at 3.25 MHz and 20 µs. This “hot spot” seems to be characteristic of the commercial transducer shown in Figure 8. Note that this highly concentrated energy density does not appear in the dispersion curves for a single laser signal. Furthermore, note that the piston profile shows higher energy density between 0.5 MHz and 1 MHz at 17 µs as compared to the Gaussian profile. This fact indicates that using a Gaussian profile instead of a piston profile improves the model, since the energy density of the commercial transducer is low below frequencies of 3 MHz. Figure 9 shows the result obtained using the inverted Gaussian window, which results in broader and more “broken up” dispersion curves (note the dark spots). The energy density for this case is high for frequencies below 3 MHz, but relatively low for higher frequencies. This phenomenon also appears for the dispersion curves of the piezoelectric disc. To justify the choice of an inverted Gaussian weight distribution as a model for the piezoelectric disc, consider the fact that the piezoelectric disc is glued to the plate. The piezoelectric disc is excited in the center and the mechanical force then propagates to the outermost regions of the disc. Since the piezoelectric disc is glued to the surface of the specimen, the excitation impulse is introduced into the specimen surface at these outermost regions. If the mechanical source is not glued, but is instead oil coupled as for the commercial transducer, this energy will not be transmitted into the plate, i.e., this mechanical energy simply propagates back and forth in the transducer and dissipates. This conclusion is corroborated by the finite element model results of Duquenne et al. [3], for calculations of the normal stress of a bonded rectangular piezoelectric element.
SUMMARY AND CONCLUSIONS

The modeling approach presented in this paper shows that the high fidelity laser source-receiver data is useful for modeling and understanding the spatial and temporal
effects of finite area piezoelectric sources in terms of recovering dispersion curves of plate-like structures. There are significant differences between the experimentally obtained dispersion curves of a bonded piezoelectric disc and that of an oil coupled commercial transducer. As expected, a band limited source time function results in a band limited version of the dispersion curves. It is further shown that the spatial effects due to a finite area source include redistributing the energy in the frequency domain, broadening the dispersion curves, and increasing the signal to noise ratio. A commercial transducer source with a laser interferometric receiver can yield useful dispersion curves although not as well-defined as those obtained with laser-laser signals.

ACKNOWLEDGEMENT

Kreuzinger thanks the DAAD (German Academic Exchange Service) for its generous financial support.

REFERENCES