GUIDED WAVE LOCALIZATION OF DAMAGE VIA SPARSE RECONSTRUCTION

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ABSTRACT. Ultrasonic guided waves are frequently applied for structural health monitoring and nondestructive evaluation of plate-like metallic and composite structures. Spatially distributed arrays of fixed piezoelectric transducers can be used to detect damage by recording and analyzing all pairwise signal combinations. By subtracting pre-recorded baseline signals, the effects due to scatterer interactions can be isolated. Given these residual signals, techniques such as delay-and-sum imaging are capable of detecting flaws, but do not exploit the expected sparse nature of damage. It is desired to determine the location of a possible flaw by leveraging the anticipated sparsity of damage; i.e., most of the structure is assumed to be damage-free. Unlike least-squares methods, L1-norm minimization techniques favor sparse solutions to inverse problems such as the one considered here of locating damage. Using this type of method, it is possible to exploit sparsity of damage by formulating the imaging process as an optimization problem. A model-based damage localization method is presented that simultaneously decomposes all scattered signals into location-based signal components. The method is first applied to simulated data to investigate sensitivity to both model mismatch and additive noise, and then to experimental data recorded from an aluminum plate with artificial damage. Compared to delay-and-sum imaging, results exhibit a significant reduction in both spot size and imaging artifacts when the model is reasonably well-matched to the data.

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INTRODUCTION

Interest is growing in guided waves as a means of damage interrogation in various structures. One common approach is to arrange simple piezoelectric discs in a spatially distributed array and visualize differences from the damage-free structure via an imaging method. The most well-known method, delay-and-sum imaging [1], is conceptually simple and utilizes little \textit{a priori} information, but its performance is limited in many cases. More advanced algorithms have been shown to achieve better performance by incorporating \textit{a priori} information about both the scatterer and propagation environment [2]. In contrast, a technique that leverages the assumption of damage \textit{sparsity} is presented here. By assuming a structure to be mostly damage free, it is possible to use sparse reconstruction techniques to achieve results superior to delay-and-sum imaging.
THEORY

Lamb Wave Propagation

One unavoidable characteristic of Lamb waves is that they are dispersive, with different frequency components traveling at different velocities. For a point source of Lamb waves, the far-field amplitude decays inversely proportionally to the square root of the distance. For convenience, a propagation operator is defined that incorporates both effects to describe a frequency-domain waveform \( F[\omega] \) after propagating a distance \( d \),

\[
P \{ F[\omega]; c_p[\omega], d \} = |d|^{-\text{sign}(d)} F[\omega] \exp \left( \frac{-i o d}{c_p[\omega]} \right).
\]

Here \( c_p[\omega] \) is the phase velocity dispersion curve for the Lamb wave mode of interest and \( d \) is the distance of propagation, with negative \( d \) indicating reverse propagation. Suppose a single-mode Lamb wave is excited at location \( \tilde{s} \) with discrete frequency-domain excitation \( F[\omega] \), and suppose the resulting waveform is recorded at location \( \tilde{p} \). This direct-arrival measurement, neglecting edge reflections, can be expressed as,

\[
Z_{\text{direct}}[\omega, \tilde{p}] = P \{ F[\omega]; c_p[\omega], \|\tilde{p} - \tilde{s}\| \}.
\]

The presence of a scatterer induces a secondary scattered wave \( Z_{\text{scattered}}[\omega, \tilde{p}] \), whose wavefield is added to the primary wave. The complex scattering pattern, denoted here by \( H[\omega, \theta_{\text{in}}, \theta_{\text{out}}] \), varies with incoming and outgoing angle as well as frequency, and depends on the shape of the scatterer. The secondary wave generated by a scatterer located at \( \tilde{p}_f \), with the source and excitation described above, is,

\[
Z_{\text{scattered}}[\omega, \tilde{p}] = H[\omega, \theta_{\text{in}}, \theta_{\text{out}}] : \mathcal{P}\left\{ Z_{\text{direct}}[\omega, \tilde{p}_f]; c_p, \|\tilde{p} - \tilde{p}_f\| \right\}
\]

\[
= H[\omega, \theta_{\text{in}}, \theta_{\text{out}}] : \mathcal{P}\left\{ F[c_p, \|\tilde{p}_f - \tilde{s}\|]; c_p, \|\tilde{p} - \tilde{p}_f\| \right\},
\]

where the incoming and outgoing angles are \( \theta_{\text{in}} = \angle (\tilde{p}_f - \tilde{s}) \) and \( \theta_{\text{out}} = \angle (\tilde{p} - \tilde{p}_f) \), respectively. Thus the actual measurement is,

\[
Z_{\text{measured}}[\omega, \tilde{p}] = Z_{\text{direct}}[\omega, \tilde{p}] + Z_{\text{scattered}}[\omega, \tilde{p}].
\]

Although this expression neglects reflections from boundaries and other geometrical features, it may be sufficiently accurate if those reflections are small in magnitude.

Sparse Reconstruction

The intent of this work is to leverage the sparse nature of damage to improve imaging results. For an operational structure (i.e., one which is pre-failure), it is reasonable to assume that most of the structure is damage-free; this is the sparsity assumption.
The typical *sparse reconstruction* problem is the recovery of a sparse vector \( \hat{x} \) from its projection \( \tilde{y} \), where \( \tilde{y} \) satisfies the linear relation,

\[
\tilde{y} = A\hat{x},
\]

for a given *dictionary* matrix \( A \). Minimization of the \( \ell_1 \) norm is one common class of optimization problem used for performing sparse reconstruction. The objective function contains an \( \ell_1 \) norm term, denoted by \( \| \cdot \|_1 \) and defined as \( \| \hat{x} \|_1 = |x_1| + |x_2| + \cdots + |x_N| \). As opposed to the more well-known \( \ell_2 \) norm, \( \| \hat{x} \|_2 = \sqrt{x_1^2 + x_2^2 + \cdots + x_N^2} \), constrained minimization of a vector’s \( \ell_1 \) norm emphasizes sparse solutions, i.e., solutions in which most components of the vector are zero-valued [3–5].

The \( \ell_1 \)-minimization technique used for this work, *basis pursuit denoising*, was selected because it offers a trade-off between accuracy and sparsity [6]. The basis pursuit denoising optimization problem is of the form,

\[
\hat{x} = \min_{\hat{x}} \frac{1}{2} \| A\hat{x} - \tilde{y} \|_2^2 + \lambda \| \hat{x} \|_1,
\]

where \( \tilde{y} \) is the measurement vector, \( A \) is the dictionary matrix of vectors whose span (approximately) contains \( \tilde{y} \), and \( \lambda \) is a regularization parameter. Small values of \( \lambda \) result in smaller errors at the expense of sparsity, whereas larger values of \( \lambda \) yield sparser solutions with larger errors. The MATLAB software package \texttt{l1_ls}*, available under the GPL v2 license, was used for this study [7].

**METHODOLOGY**

It is necessary to formulate structural imaging as a linear optimization problem to exploit the sparsity assumption and use sparse reconstruction. This formulation requires modeling the received signal, \( \tilde{y} \), and generating the dictionary, \( A \). By performing the optimization described in Eq. (6), the sparse vector \( \hat{x} \) may be recovered; non-zero entries should correspond to potential damage locations.

**Received Signal Model**

For the problem setup used in this paper, a complete set of pairwise signals is recorded from a sparse array of \( n \) transducers to constitute a measurement; thus each measurement contains \( m = n(n - 1)/2 \) signals. A baseline measurement, \( b_i[t] \), \( 1 \leq i \leq m \), is performed, after which scatterers are introduced and a second measurement is taken, \( s_i[t] \), \( 1 \leq i \leq m \).

Equation (6) requires the measurement \( \tilde{y} \) to be a vector, and it must be linear in \( \hat{x} \). If second-order scattering (scatterer-to-scatterer interactions) are neglected, it is clear that residual signals, \( r_i[t] = (s_i[t] - b_i[t]) \), are linear in scatterer placement; i.e., the residual signal due to two scatterers is (approximately) the sum of the residual signals due to each scatterer individually. Since Eq. (6) requires a vector measurement, the individual signals
must be vectorized by assigning the variable $\tilde{y}$ to be the concatenation of these residual signals into a column vector:

$$
\tilde{y} = [r_1^T \quad r_2^T \quad \ldots \quad r_m^T]^T.
$$

(7)

This process is graphically shown in Figure 1.

**Dictionary Formulation**

The dictionary $A$ must reflect the sparsity assumption inherent in $\tilde{x}$. Each column corresponds to a particular point on the area of interest. This area is discretized into pixel locations, $\tilde{p}_k$, $1 \leq k \leq P$; thus the length of $\tilde{x}$ (and, therefore, the number of columns in $A$) is equal to $P$, since each entry of $\tilde{x}$ corresponds to one pixel. Nonzero $x_k$ should correspond to the presence of a scatterer at location $\tilde{p}_k$.

The assumption that damage, and therefore $\tilde{x}$, is sparse, coupled with Eq. (5), implies that $\tilde{y}$ is the linear combination of a small number of columns of $A$, and that furthermore each of these columns represents the contribution to $\tilde{y}$ from a scatterer at a location that corresponds to its column index. Thus, each column of $A_k$ should be the expected measurement (i.e., $\tilde{y}$) in the event that only one scatterer is present at $\tilde{p}_k$.

![FIGURE 1. Example measurement vector $\tilde{y}$ for four sensors. (a) Propagation paths (transmitter-scatterer-receiver) for all sensor pairs. (b) Signals acquired and shown after baseline subtraction. (c) Signals stacked to form the measurement vector $\tilde{y}$.](image-url)
To calculate the dictionary (columns of $A$), we assume a known scattering pattern $H[\omega, \theta_{in}, \theta_{out}]$, single-mode propagation with a known dispersion relation $c_p[\omega]$, and no edge reflections. For a known excitation $F[\omega]$ (expressed here in the frequency domain), Eq. (3) shows that the expected residual signal recorded by $i^{\text{th}}$ pair of sensors for a single flaw at $\vec{p}_k$ is,

$$Y_i[\omega; \vec{p}_k, H] = H[\omega, \theta_{in}, \theta_{out}].P\left\{ F; c_p, \left\| \vec{p}_k - \vec{s}_i \right\|_2; c_p, \left\| \vec{r}_i - \vec{p}_k \right\|_2 \right\},$$

(8)

where $\vec{s}_i$ and $\vec{r}_i$ are the locations of the $i^{\text{th}}$ pair’s source and receiver, respectively. In other words, the source excitation function is forward propagated from the source to $\vec{p}_k$, multiplied by the appropriate transfer function of the scatterer, and then forward propagated to the receiver. These simulated signals, after conversion to the time domain via the inverse Fourier transform, are concatenated to form a single column of $A$. Each column is then normalized to unity $\ell_2$ norm.

$$\vec{A}_k \propto \begin{bmatrix} \mathcal{F}^{-1}\{Y_1[\omega, \vec{p}_k, H]\} \\ \mathcal{F}^{-1}\{Y_2[\omega, \vec{p}_k, H]\} \\ \vdots \\ \mathcal{F}^{-1}\{Y_m[\omega, \vec{p}_k, H]\} \end{bmatrix}$$

(9)

This process is repeated for all $1 \leq k \leq P$ to generate $A$.

**Sparse Imaging**

Generating the sparse image of scatterers simply requires performing the optimization in Eq. (6), or a similar sparsity-based inversion of Eq. (5). To form a 2D image to visualize the results, the values of $\vec{x}$ are reshaped from a vector into an image so that each pixel $\vec{p}_k$ has a value $x_k$. Non-zero pixel values may be suggestive of damage.

**Scatterer Considerations**

The methodology discussed requires knowledge of a potential scatterer’s scattering pattern. While this knowledge may be available in some circumstances where a particular type of damage is expected, there may be situations where a priori scattering information is unavailable or incomplete, particularly phase information of the scatterer transfer function. In these cases, performance may be improved by using amplitude envelopes for $\vec{y}$ and $A$, and using $H[\omega, \theta_{in}, \theta_{out}] = 1$. Using envelope detection on the differenced signals and dictionary entries will eliminate the (possibly severe) effects of scatterer phase mismatch. However, since envelope detection is not a linear operation, this comes at the cost of degradation of the linear assumption inherent in Eq. (5). In practice, however, this non-linearity may have a smaller effect on performance than incorrect phase assumptions.
FIGURE 2. Sparse imaging results from simulated data for three scatterers shown on a 20 dB intensity scale. (a) Full view; actual scatterer locations are in the centers of the squares. (b) (c) (d) Zoomed views of the sparse imaging results where the open triangles denote the actual scatterer locations.

NUMERICAL RESULTS

Initial testing of the methodology was performed using numerical simulation of several configurations of multiple uniform point scatterers on a 3.175 mm thick Al-6061 plate with eight transducers. The excitation was a 100 kHz, 5-cycle Hann-windowed tone burst, and the propagation model assumed a pure $A_0$ Lamb wave mode. Multiple scattering and edge reflections were not modeled, and envelope detection was not used. Thus, this simulation reflects a “best case” scenario and is thus valuable for assessing the upper limit of performance. Results are shown in Figure 2 for a dictionary resolution of 4 mm and, as expected, are excellent. For scatterers located exactly at a pixel location, only that pixel is assigned a value; for scatterers between pixels, adjacent pixels are nonzero.

EXPERIMENTAL RESULTS

A 3.175 mm thick, 1220 mm x 1220 mm Al-6061 plate was used for experimental purposes. For the initial study, it was desired to reduce the effects of edge reflections, so in addition to the large dimensions of the plate, the edges were further damped with duct-sealing compound. Eight 0.5 mm thick, 7 mm diameter PZT discs were affixed in a roughly circular pattern. Cylindrical masses were glued to the plate to simulate damage, and scattering was assumed to be uniform. For generation of the $A$ matrix, model-based parameter estimation was performed on the baseline signals as described in [8] to estimate $c_p(\omega)$ and transducer transfer functions. A broadband excitation was used and then post-processed as described in [9] to obtain the equivalent response to a 100 kHz, 5-cycle Hann-windowed tone burst, which has a dominant $A_0$ mode for this plate thickness.

Figure 3 shows both delay-and-sum and sparse reconstruction results for a single scatterer using envelope-detected signals. Since the edge reflections do not overlap with the scattered echoes, the delay-and-sum image shows an unambiguous (but spatially large) indication of the scatterer. For the sparse reconstruction results, all non-zero pixels are in the immediate vicinity of the scatterer location and exhibit the expected spatial sparsity. Figure 4 shows corresponding results for two scatterers and results are similar. The scatterers are far enough apart so that they can be resolved on the delay-and-sum image. On the sparse reconstruction image the scatterers are completely separated, and no pixels away from the scatterer locations are large enough to be seen on the 20 dB scale used here.
FIGURE 3. Experimental imaging results for a single scatterer shown on a 20 dB intensity scale. (a) Delay-and-sum image where the solid triangle denotes the actual scatterer location. (b) Sparse imaging results where the actual scatterer location is in the center of the square. (c) Zoomed version of the sparse imaging results where the solid triangle denotes the actual scatterer location.

FIGURE 4. Experimental imaging results for two scatterers shown on a 20 dB intensity scale. (a) Delay-and-sum image where the solid triangles denote the actual scatterer locations. (b) Sparse imaging results where the actual scatterer locations are in the center of the squares. (c) Zoomed versions of the sparse imaging results where the solid triangles denote the actual scatterer locations.

SUMMARY AND CONCLUSIONS

This paper has demonstrated feasibility of using sparse reconstruction methods to produce accurate and highly localized images of scatterers for the application of damage detection using differenced signals. This work is significant because it explicitly incorporates the expected sparsity of damage into the problem formulation. It also has the potential to characterize scatterers by including additional scattering models in the dictionary, and to greatly reduce data acquisition and storage requirements by incorporating compressed sensing. Future work should include better scattering models so that phase can be used, methods to quantify damage severity, evaluation of more computationally efficient solution methods, and further experiments in more complicated specimens where there may be significant model mismatch.
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REFERENCES


