INCIDENT WAVE REMOVAL THROUGH FREQUENCY-WAVENUMBER FILTERING OF FULL WAVEFIELD DATA

Thomas E. Michaels¹, Massimo Ruzzene² and Jennifer E. Michaels³

¹School of Electrical and Computer Engineering, Georgia Institute of Technology, Atlanta, GA 30332-0250
²School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, GA 30332-0150

ABSTRACT. Full wavefield measurements of guided waves in a structure provide a wealth of information that can be effectively used for the identification, localization and quantification of damage. Full field measurements can be obtained using an air-coupled transducer mounted on a scanning stage, or via a scanning laser vibrometer. The resulting detailed temporal and spatial information can be transformed to the frequency-wavenumber domain where waves propagating in different directions appear decoupled. Appropriate filtering strategies can be applied to effectively remove the contribution of incident waves while highlighting reflections and scattering associated with structural discontinuities or damage. This paper presents two frequency-wavenumber domain analysis methods. The first method is based upon multiple two-dimensional Fourier Transforms applied to waveform data in spatial polar coordinates, and the second applies the three-dimensional Fourier Transform in Cartesian coordinates. The utility of these methods is demonstrated on full wavefield data recorded on a composite plate before and after the introduction of damage.

Keywords: Structural Health Monitoring, Acoustic Wavefield Imaging, Guided Waves, Composites

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INTRODUCTION

Guided wave methods have been proposed for structural health monitoring that use permanently attached transducers to send and receive waves that interrogate a structure for damage [1,2]. Actuation of these attached transducers generates an outward propagating acoustic wavefield from the source that is measurable on the surface of the structure, using either an externally scanned transducer or a scanning laser vibrometer [3,4]. Wavefield images are thus obtained as a set of RF waveforms recorded over a spatial grid on the surface of the structure. As waves propagate out from the source, interactions with structural discontinuities are directly visible on snapshots of the wavefield, which typically show the outward propagating incident wave, waves scattered from impedance...
discontinuities, and reflections from structural boundaries [5,6]. Signals scattered from defects are typically much smaller in amplitude than the incident waves. Thus, it is desirable to remove incident waves from wavefield images to enhance waves scattered from defects. In previous work, several methods were investigated to enhance damage visualization in a structure using acoustic wavefield images [7,8]. This paper introduces a new two dimensional frequency-wavenumber domain filtering method, implemented in polar coordinates, for enhancement of wavefield images. Three dimensional frequency-wavenumber methods are also applied, similar to those described in [8]. Results are shown for propagation of waves in an anisotropic graphite composite plate specimen.

EXPERIMENTAL METHODS

A graphite epoxy plate, 305 mm x 305 mm x 3.2 mm and with lay-up orientations of [0/90/0/90/0], was used for this study. The propagating acoustic wavefield in the plate was measured using a QMI AS400ARi 400 MHz air-coupled transducer attached to an automated scanner. The wave source was a 2.25 MHz x 12.7 mm diameter piezoelectric disk, which was permanently attached to the back surface using a semi-rigid epoxy adhesive. The ultrasonic instrument was an Olympus PR5058 pulser receiver. The plate was raster scanned at a resolution of 1.27 mm, and RF waveforms were digitized and stored at a sampling frequency of 2.5 MHz x 8 bits at each pixel location on the plate.

Time slice snapshots of the propagating wavefield are shown in Figure 1 after damage was added to the plate. Waves are visible radiating out from the source transducer at (85, 65), where these and subsequent (x, y) coordinates are given in mm. A delamination was intentionally added to the plate by impacting it with a hammer at (189, 95), and scattered waves are visible radiating outward from this location. Scattering and mode conversion are also visible from a second, passive piezoelectric disc attached to the back of the plate at (135, 183). Reflections from the edges of the plate are also directly visible on these images.

![Figure 1](https://via.placeholder.com/150)

**FIGURE 1.** Snapshots of the ultrasonic wavefield radiating from a 2.25 MHz source transducer permanently attached to a composite plate specimen.
ANALYSIS METHODS

For the analysis results that follow, the measured wavefield in Cartesian coordinates in the time domain is referred to as \( u(x, y, t) \).

2D Frequency Wavenumber Domain Analysis

The following steps are used for filtering and analysis of waveform data \( u(x, y, t) \) in the two-dimensional frequency-wavenumber domain, \( \omega - k_r \).

- The source location \((x_s, y_s)\) is found from the location of first wave activity.
- Time domain waveform data are transformed from \( u(x, y, t) \) in Cartesian coordinates to \( u(r, \theta, t) \) in polar coordinates, centered at \((x_s, y_s)\).
- Waves along a radial line from the source, denoted as \( u(r, \theta, t) \), where \( \theta_i \) is the specific angle index, are individually transformed to the \( \omega - k_r \) domain as
  \[
  U(\omega, k_r; \theta_i) = F_{2D}[u(r, \theta, t)],
  \]
  where \( F_{2D} \) is the two-dimensional Fourier Transform.
- Filtering is applied to \( U(\omega, k_r; \theta_i) \) in the frequency-wavenumber domain by
  \[
  \tilde{U}(\omega, k_r; \theta_i) = U(\omega, k_r; \theta_i) \cdot W(\omega, k_r),
  \]
  where \( W(\omega, k_r) \) is the filter. Filtering in the frequency-wavenumber domain is used to separate modes, i.e., \( S_0 \) from \( A_0 \), and propagation directions, i.e., forward (outgoing) waves from backward (incoming) waves.
- Filtered frequency wavenumber data \( \tilde{U}(\omega, k_r; \theta_i) \) are transformed back to the time domain to give
  \[
  \tilde{u}(r, \theta_i, t) = (F_{2D})^{-1} \{ \tilde{U}(\omega, k_r; \theta_i) \},
  \]
  where \((F_{2D})^{-1}\) is the inverse two-dimensional Fourier Transform.
- Finally, for all values of \( \theta_i \), a polar to Cartesian coordinate transformation is used to obtain \( \tilde{u}(x, y, t) \), the filtered data in the time domain in Cartesian coordinates.

3D Frequency Wavenumber Domain Analysis

The following steps are used for analysis of waveform data in the three-dimensional frequency-wavenumber domain, \( \omega - k_x - k_y \).

- Time domain waveform data \( u(x, y, t) \) or \( \tilde{u}(x, y, t) \) are windowed over a spatial region of interest to obtain \( u_w(x, y, t) \).
- Windowed waveform data \( u_w(x, y, t) \) are transformed to the \( \omega - k_x - k_y \) domain by
  \[
  U_w(\omega, k_x, k_y) = F_{3D}[u_w(x, y, t)],
  \]
  where \( F_{3D} \) is the three-dimensional Fourier Transform.
- Frequency-wavenumber domain data \( U_w(\omega, k_x, k_y) \) is windowed in frequency to produce \( \tilde{U}_w(\omega, k_x, k_y) \), where frequencies are restricted to the range \( \omega_1 \) to \( \omega_2 \).
- Two dimensional \( k_x - k_y \) regions of \( \tilde{U}_w(\omega, k_x, k_y) \) are analyzed to show wave modes, propagation directions, and changes in the scattered wavefield at damage sites.
RESULTS

Steps of the 2D frequency-wavenumber analysis method are illustrated in Figure 2. Figure 2(a) is a wavefield snapshot showing the source location. Figure 2(b) is a snapshot of the wavefield at 66 μs, and Figure 2(c) shows the transformation to polar coordinates. The two-dimensional frequency-wavenumber plot for a 45° radial line from the source is shown in Figure 2(d), and forward propagating wave modes $S_0$ and $A_0$ are visible on this plot. Figure 3 is the frequency-wavenumber plot for propagation along the negative $y$ direction, -90°, and four wave modes, both forward and backward $S_0$ and $A_0$, are visible on this plot. Forward propagating waves move away from the source and correspond to positive wavenumber values. Backscattered waves from the flaws and edge reflections correspond to negative wavenumber values.

FIGURE 2. Two-dimensional frequency-wavenumber analysis steps. (a) Snapshot used to find the source location, (b) wavefield snapshot in Cartesian coordinates, (c) wavefield snapshot in polar coordinates, and (d) two-dimensional Fourier transform of the radial line of waveforms at an angle of 45° from 0 to 68 μs.

FIGURE 3. Wave modes in the frequency-wavenumber domain at an angle of -90°.
The 2D frequency-wavenumber filtering and analysis procedure is first used to remove the incident S\textsubscript{0} mode from the wavefield. Figure 4(a) is a snapshot of the wavefield for a propagation time of 58 \(\mu\)s after first source activity, Figure 4(b) shows a frequency-wavenumber slice after removal of the forward S\textsubscript{0} mode, and Figure 4(c) shows the snapshot of the filtered wavefield after transformation back to the time domain. Figures 4(d) and 4(e) illustrate removal of all forward waves, both S\textsubscript{0} and A\textsubscript{0}. Both of the impedance discontinuities — the delamination and the passive transducer mounted on the back of the plate — are directly visible on these filtered wavefield images. The removal of the incident wavefield has made it possible to directly visualize scattered waves from these sites.

As is apparent in Figure 1, the S\textsubscript{0} mode propagates fastest along the x and y directions. Between these principal directions the wave front propagates at an angle of approximately 45°, and the angle of the \(k\) vector in the \(k_x-k_y\) domain lies along this propagation direction. To illustrate, the wavefield within the box shown in Figure 5(a) is analyzed using the 3D Fourier transform in the frequency range shown in Figure 5(b). Results are shown in Figures 5(c) and 5(d). Note that the \(k\) vector lies along the propagation direction as expected. These figures contain frequency components from 0.1 MHz to 0.6 MHz, and the energy is spread over a substantial range in \(k\) space. A narrower frequency pass band would result in a more localized region on these plots.

Next, the boxed region around the source transducer, as shown in Figure 6(a), is examined in the \(k_x-k_y\) domain. Figure 6(b) is the \(k_x-k_y\) domain plot over a broad frequency range, from 0.1 to 0.6 MHz. This range includes most of the source energy, but is so large that individual modes are obscured. Figure 6(d) shows results over the narrow frequency range depicted in Figure 6(c), 0.24 to 0.26 MHz, from which individual S\textsubscript{0} and A\textsubscript{0} modes may now be identified.

![FIGURE 4. Removal of wave components in the frequency-wavenumber domain.](image)

(a) Snapshot of the unfiltered wavefield at 58 \(\mu\)s, (b) removal of forward S\textsubscript{0} waves in the frequency-wavenumber domain, (c) wavefield after forward S\textsubscript{0} waves are removed, (d) removal of all forward waves in the frequency-wavenumber domain, and (e) wavefield at 58 \(\mu\)s after all forward waves are removed.
FIGURE 5. Three dimensional frequency wavenumber results. (a) Region of wavefield selected for analysis, (b) frequency spectrum for a RF waveform at center of this selected region, (c) results in the $k_x-k_y$ domain, and (d) isometric plot of $k_x-k_y$ domain results.

FIGURE 6. Analysis of the wavefield source from a 2.25 MHz transducer permanently attached to a composite plate. (a) Spatial region analyzed above the source transducer, (b) results in the $k_x-k_y$ domain for a broad frequency range (0.1 to 0.6 MHz), (c) frequency spectrum of the waveform from the location directly above the source, (d) results in the $k_x-k_y$ domain for a narrow frequency range (0.24 to 0.26 MHz).
Plots in the $k_x-k_y$ domain are also useful for examining details of the wavefield scattered from discontinuities. Figure 7 shows results of using this type of analysis for the area where a delamination was induced. First, all forward waves were removed by 2D frequency-wavenumber domain filtering of the wavefield in polar coordinates, and the resultant forward filtered wavefield results are shown in Figures 7(a) and 7(d), before and after a delamination was added to the plate. Next, the boxed area, which encloses the delamination site, was transformed to the $k_x-k_y$ domain. The utility of the $k_x-k_y$ domain for analyzing the nature of the scattered field is shown in Figures 7(e) and 7(f), where $k$ vector components are shown for waves backscattered from the delamination. Compared to Figures 7(b) and 7(c), before damage was induced, there is a substantial increase in the scattered wavefield. All the energy in the scattered wavefield is localized in the $k_x-k_y$ domain, whereas it spreads out in the time domain as waves propagate away from this scattering site. Thus, the $k_x-k_y$ domain is useful for calculating the total energy of waves scattered from a defect. Although beyond the scope of this paper, accumulated energy can be related to a damage index, which can be directly computed in the $k_x-k_y$ domain [9].

**SUMMARY AND CONCLUSIONS**

The utility of frequency-wavenumber domain analysis has been demonstrated for enhancing the visualization of damage using acoustic wavefield images. A method was introduced for filtering acoustic wavefield data in a two dimensional frequency-wavenumber domain. Time domain waveforms collected over a Cartesian pixel grid are transformed to polar coordinates centered at the source transducer location, and each angular $(r,t)$ slice is transformed to the frequency-wavenumber domain.
Forward and backward propagating waves are decoupled in the frequency-wavenumber domain, and filtering procedures are used to separate various wave modes. Using this filtering method, it is possible to completely remove the strong incident waves propagating away from the source transducer, thereby visually enhancing waves that are backscattered from defects and allowing for their identification and interpretation.

Analysis of wavefield data in the 3D frequency-wavenumber domain is useful for determining the propagation direction and symmetry of various modes. This method was used to decouple individual $S_0$ and $A_0$ modes generated by the source transducer, thereby directly showing the anisotropic propagation of $S_0$ waves in a composite plate specimen. Analysis in the 3D frequency-wavenumber domain is also useful for detailed investigation of waves backscattered from defects.

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