ONE-SIDED LIQUID COUPLED ULTRASONIC METHOD FOR RECOVERY OF THIRD ORDER ELASTIC CONSTANTS

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ABSTRACT. Ultrasonic measurements for experimentally determining third order elastic constants (TOECs) typically utilize multiple setups for bulk waves with directions of propagation parallel and perpendicular to the direction of applied stress. Experiments can be tedious and may require access to both sides of a specimen, which can be problematic in field applications for propagation parallel to the loading direction. Shown here is an alternative ultrasonic technique which utilizes an angle beam method whereby transducers are mounted on the same side of a parallel faced specimen. The general theory of acoustoelasticity for homogeneous and isotropic materials is used to derive acoustoelastic constants for propagation at an angle to the principal stress directions. TOECs are then estimated utilizing one shear vertical and two longitudinal angle beam measurements in an oil coupled or floating configuration. Experimental results are shown for aluminum 7075 and compared to published TOEC values. A sensitivity analysis is performed to understand how small variations in all input parameters affect the accuracy of the recovered third order elastic constants.

Key words: Acoustoelasticity, Third Order Elastic Constants, Ultrasonic Angle Beam

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INTRODUCTION

Third order elastic constants (TOECs) of a material are directly related to the acoustoelastic effect, which refers to the change in the speed of propagation of an elastic wave as a function of applied load. An acoustoelastic constant $K$ can be defined for a particular experiment as,

$$K = \frac{\Delta c}{c \sigma},$$

where $c$ is the wave speed and $\sigma$ is the applied stress. The Murnaghan theory of finite deformations provided the basis for the acoustoelasticity work performed by Hughes and Kelly in 1953 [1], and Pao et al. published an extensive review in 1984 [2].

Measurements of wave speed as a function of stress are one of the few approaches for determining TOECs, which describe non-linear deviations of the stress-strain constitutive relationship of a material. This non-linearity has also been studied by measuring higher
order harmonics resulting from sinusoidal excitations, and efforts have been made to correlate these harmonics to damage [3]. Direct measurement of various acoustoelastic constants has also been considered for damage detection [4, 5]. For both types of measurements, the non-linear effects are small, measurements are very tedious and time-consuming, and repeatability can be a problem.

In this paper we consider determining TOECs of an isotropic solid from angle beam acoustoelasticity measurements. This approach differs from that described in the literature in that the direction of propagation is not along any of the principal stress directions. Also, transducers are mounted in a floating configuration, i.e., they are liquid-coupled to the specimen with a fixed transducer separation distance. Even though the general theory of acoustoelasticity applies to this situation, the specific derivations for determining the TOECs from angle beam measurements of acoustoelastic constants have not been reported outside our previous work [6, 7].

There are several possible advantages of a liquid coupled angle beam technique. First, it can readily be implemented as a one-sided technique, not requiring transducers to be mounted on both sides of a specimen. Second, it is not necessary to embed transducers inside of the specimen grips to generate beam paths along the loading direction. Third, for thin specimens, time shifts are also larger than those for normal incidence measurements, possibly making the angle beam measurements more robust. Fourth, since the transducers are not glued, as was assumed in [7], the transducers and wedges are not strained. And last, mounting and alignment of transducers, although still critical, is simplified, particularly for thin specimens.

THEORY

Forward Problem

The forward problem is calculation of time shifts from the applied load and material parameters using known third order elastic constants. This problem was first considered by Mi et al. for angle beam shear wave propagation in a glued-on application [6], and later by Muir et al. for both longitudinal and shear waves [7]. A similar approach is followed here.

The measured change in time-of-flight for a given angle beam configuration and applied load \( \sigma \) depends upon both the change in wave speed (acoustoelastic effect) and the change in path due to deformation (i.e., the strain). For the general floating case, a single “V” path configuration is illustrated in Figure 1 with transducer separation distance \( 2D \) and thickness \( h \). The resulting time shift, \( \Delta T_{\text{meas}} \), is,

\[
\Delta T_{\text{meas}} = \Delta T_{\text{geometry}} + \Delta T_{\text{acoustoelasticity}} \\
\approx \left( \frac{\Delta P}{c} \right) + \left( -\frac{P \Delta \epsilon}{c^2} \right) \\
\]

FIGURE 1. Sample of thickness \( h \) with a single “V” angle beam path of refracted angle \( \theta \).
In Eq. (2) $P$ is the total path length, $c$ is the wave speed, and $\Delta P$ and $\Delta c$ are the changes in path length and wave speed, respectively, caused by the load. It is mathematically convenient to work in the natural (unstrained) coordinate system, and in this system, as the load changes, the thickness remains the same and the transducer separation distance decreases. The time shift due to geometry can thus be expressed as,

$$
\Delta T_{\text{geometry}} = \frac{2\sqrt{D^2 (1-\sigma/E)^3} + h^2 - 2\sqrt{D^2 + h^2}}{c}
$$

where $E$ is Young’s modulus $\sigma/E$ is the strain in the direction of loading.

For either shear or longitudinal waves propagating at an angle $\theta$, the effective acoustoelastic constant $K(\theta)$ due to applied uniaxial stress is of the form,

$$
K(\theta) = K_1 \sin^2 \theta + K_2 \cos^2 \theta,
$$

where $\theta$ is the refracted angle, and $K_1$ and $K_2$ are the acoustoelastic constants corresponding to $\theta = 90^\circ$ (horizontal propagation direction) and $\theta = 0^\circ$ (vertical propagation direction), respectively. The constants for longitudinal wave propagation can be derived as [7],

$$
K_1^l = \frac{4 \lambda^2 + 15 \lambda \mu + 10 \mu^2 + 2l \mu + 4 m \lambda + 4 m \mu}{2 \mu (3 \lambda + 2 \mu)(\lambda + 2 \mu)} \quad \text{and} \quad K_2^l = \frac{-\lambda^2 - 2 \lambda \mu - m \lambda + l \mu}{\mu (3 \lambda + 2 \mu)(\lambda + 2 \mu)},
$$

and corresponding constants for SV waves are [6]

$$
K_1^{sv} = \frac{4 \mu (\lambda + \mu) + m \mu + \lambda n / 4}{2 \mu^2 (3 \lambda + 2 \mu)} \quad \text{and} \quad K_2^{sv} = \frac{\mu (\lambda + 2 \mu) + m \mu + \lambda n / 4}{2 \mu^2 (3 \lambda + 2 \mu)}.
$$

In Eqs. (5) and (6), $l$, $m$ and $n$ are the Murnaghan third order elastic constants [8], and $\lambda$ and $\mu$ are the Lamé constants. The time shift due to acoustoelasticity then becomes,

$$
\Delta T_{\text{acoustoelasticity}} = -\frac{P}{c} c [\sigma K(\theta) - \varepsilon(\theta)] = \frac{P \varepsilon(\theta)}{c} - \frac{P \sigma K(\theta)}{c},
$$

where $\varepsilon(\theta)$, the strain along the direction of propagation, must be included to map the velocity from the strained system to the natural system [2,6]. For a uniaxial loading of $\sigma$, this strain is $\varepsilon(\theta) = \frac{2(\lambda + \mu) \sin^2 \theta - \lambda \cos^2 \theta}{2 \mu (3 \lambda + 2 \mu)} \sigma$. Time shifts due to both geometry and acoustoelasticity can now be calculated for a known loading and angle beam configuration.

**Inverse Problem**

The goal of the inverse problem is to determine the three TOECs from the measured time shifts ($\Delta T_{\text{meas}}$) and applied load. The time shift due to geometry does not depend on TOECs and can thus be calculated from Eq. 3. It is subtracted from $\Delta T_{\text{meas}}$ to give the time shift due to acoustoelasticity, which can be expressed as a linear function of the TOECs,

$$
\Delta T_{\text{acoustoelasticity}} = \Delta T_{\text{meas}} - \Delta T_{\text{geometry}} = a_0 + a_1 l + a_2 m + a_3 n.
$$
The approach to solving the inverse problem can best be understood by examining Eqs. (4), (5) and (6), which express the acoustoelastic constants in terms of the second and third order elastic constants. Note that constants in Eq. (5) for the L waves contain only $l$ and $m$, and similarly Eq. (6) for SV waves contain only $m$ and $n$. Furthermore, the SV equations have the same coefficients for $m$ and $n$. To determine all three constants, three equations (i.e., measurements) are required. Only one SV refracted angle can be used because the relative weights of $m$ and $n$ have no angular dependence. Thus, there must be two L measurements at two different refracted angles. The two L measurements determine $l$ and $m$, and the SV measurement determines $n$.

Once the configurations for the three measurements have been determined, the next step is to measure the corresponding time shifts for a specified applied load of $\sigma$. Eqs. (4), (5) and (6) for the L and SV waves are combined to yield a set of three linear equations,

$$
\begin{bmatrix}
  a_1 & a_2 & a_3 \\
  b_1 & b_2 & b_3 \\
  c_1 & c_2 & c_3
\end{bmatrix}
\begin{bmatrix}
  l \\
  m \\
  n
\end{bmatrix}
=
\begin{bmatrix}
  \Delta T_{\text{mean}}^{SV} - \Delta T_{\text{geometry}}^{SV} - a_0 \\
  \Delta T_{\text{mean}}^{L1} - \Delta T_{\text{geometry}}^{L1} - b_0 \\
  \Delta T_{\text{mean}}^{L2} - \Delta T_{\text{geometry}}^{L2} - c_0
\end{bmatrix}
$$

(9)

The coefficients $a_j$, $b_j$ and $c_j$ coefficients are functions of $\lambda$, $\mu$, $h$, $D$ (all three values) and $\sigma$. Note that $a_1$, $b_3$ and $c_3$ are all zero. This linear system of equations can then be solved for the three TOECs $l$, $m$ and $n$.

EXPERIMENTAL PROCEDURE

The acoustoelastic measurement process consists of measuring time shifts of received ultrasonic echoes as a function of the applied uniaxial load for a specific angle beam configuration. Figure 2(a) shows the measurement setup, where the specimen is held in a loading fixture designed to achieve uniaxial tensile loading via a manual hydraulic pump and cylinder. The assembly is fitted with a pressure gauge and load cell, which have both been calibrated to determine the applied load. As shown in Figure 2(b), two 5 MHz angle beam transducers are attached to various fixed wedge pairs designed to generate the desired refracted angles. The transducer/wedge assembly is then clamped onto one side of the specimen after being coupled with oil. The assembly is positioned and aligned parallel to the direction of uni-axial loading. The wedge pair fabrication is based upon calculated distances for the mode, refracted angle, and number of “V” paths required. Table 1 gives a summary of the four configurations used for the measurements. A conventional spike-mode pulser-receiver is used in the through transmission mode, and the received signals are digitized at a sampling frequency of 5 GHz before further processing.

![Figure 2](image)

**FIGURE 2.** (a) Angle beam measurement setup showing one of the transducers/wedge assemblies clamped to the aluminum sample. (b) Wedge pairs used for the angle beam measurements.
TABLE 1. Four angle beam measurement configurations for an aluminum sample 6.26 mm thick.

<table>
<thead>
<tr>
<th>Setup</th>
<th>Mode</th>
<th>V-Paths</th>
<th>$\theta$ (degrees)</th>
<th>2D (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>SV</td>
<td>1</td>
<td>45</td>
<td>12.52</td>
</tr>
<tr>
<td>2</td>
<td>SV</td>
<td>1</td>
<td>70</td>
<td>34.42</td>
</tr>
<tr>
<td>3</td>
<td>L</td>
<td>1</td>
<td>45</td>
<td>12.52</td>
</tr>
<tr>
<td>4</td>
<td>L</td>
<td>1</td>
<td>60</td>
<td>21.69</td>
</tr>
</tbody>
</table>

Time shift measurements were made as follows: (1) Record the waveform from the unloaded specimen, (2) quickly load the specimen to a precalculated maximum, (3) record the actual load and waveform, (4) release the load, and (5) record the waveform at the nominal zero-load state. Steps 2-5 are then repeated four more times in succession, resulting in a total of nine stored waveforms. Time shifts are calculated to a resolution of 0.025 ns from the interpolated cross correlation between successive waveforms, yielding eight time shifts (four for loading and four for unloading). Experience has shown that the unloading segments yield the most consistent and repeatable data, so these four time shifts are averaged to obtain the final time shift. The accuracy of the time shifts is estimated to be about ±1 ns, and the accuracy of the force applied is approximately ±500 N.

RESULTS

The process of determining TOECs was carried out experimentally by measuring load-dependent time shifts for the four angle beam configurations of Table 1. Measured and calculated time shifts for the forward problem are compared, and the recovered TOECs are compared to literature values. Finally, a sensitivity analysis was performed to evaluate the effect of measurement errors in all input parameters on the recovered TOECs.

Experimental Results

Experimental results are presented here for the forward problem of determining time shifts and the inverse problem of determining TOECs. The specimen was a 7075 aluminum coupon with a thickness of 6.26 mm and a width of 76.3 mm. The reference load considered was 194 MPa, which is well below the yield strength of the material. Constants $l$, $m$ and $n$ were previously characterized for 7075-T651 by Stobbe [9] using conventional ultrasonic methods (i.e., beam paths perpendicular to the loading direction) and are shown in Table 2 along with other TOECs for aluminum that have been reported in the literature [10, 11]. Values for the Lamé constants ($\lambda$ and $\mu$) and density values were taken to be 54.9 GPa, 26.5 GPa and 2800 kg/m³, respectively, and other constants were computed from these (e.g., $E$, $c_L$, and $c_S$).

Figure 3(a) shows measured time shifts compared to those calculated using TOECs for 7075 reported by Stobbe [9] and Dubuget et al. [10]. The trends of the time shifts are in good agreement, although the measured values are smaller than the calculated ones except for the 45° shear wave configuration. Figure 3(b) compares experimentally recovered TOECs to those of Stobbe [9] and Dubuget et al. [10]. Note that the experimental value for $n$ is the average from the two SV measurements.
TABLE 2. Summary of experimentally determined TOECs and literature values.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>l</td>
<td>-65.2</td>
<td>-252.2</td>
<td>-126 ± 15</td>
<td>-103 ± 15</td>
<td>-311</td>
</tr>
<tr>
<td>m</td>
<td>-292.6</td>
<td>-325.0</td>
<td>-320 ± 10</td>
<td>-279 ± 10</td>
<td>-401</td>
</tr>
<tr>
<td>n</td>
<td>-314.4</td>
<td>-351.2</td>
<td>-282 ± 6</td>
<td>-333 ± 6</td>
<td>-408</td>
</tr>
</tbody>
</table>

FIGURE 3. (a) Time shift data for four transducer configurations at maximum load. (b) Recovered TOECs using time shift data of (a) and compared to literature values.

Sensitivity Analysis

The sensitivity analysis considers the effect of errors in all measurements and parameters on recovered TOECs. Numerical results were obtained using one SV and two L configurations as summarized in Table 3. Times-of-flight were computed based upon a load of 194 MPa using the procedure outlined in the theory section and with constants for 7075-T651 as determined by Stobbe [9]. Equation (8) was then solved to recover l, m and n. Details of the errors and nominal values are shown in Table 3, and Figure 4 shows the results in terms of the effect of the estimated uncertainty of each parameter on the uncertainty of each TOEC. Column 13 in the figure shows the total uncertainty of each TOEC.

TABLE 3. Input parameters for uncertainty analysis of TOEC recovery method.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Nominal Value</th>
<th>Uncertainty</th>
<th>Parameter</th>
<th>Nominal Value</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Time Shift, 70S</td>
<td>28.18 ns</td>
<td>1 ns</td>
<td>7. D, 45L</td>
<td>6.26 mm</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>2. Time Shift, 45L</td>
<td>7.93 ns</td>
<td>1 ns</td>
<td>8. D, 60L</td>
<td>10.85 mm</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>3. Time Shift, 60L</td>
<td>24.11 ns</td>
<td>1 ns</td>
<td>9. Lambda</td>
<td>54.9 GPa</td>
<td>2 GPa</td>
</tr>
<tr>
<td>4. Thickness</td>
<td>6.26 mm</td>
<td>0.08 mm</td>
<td>10. Mu</td>
<td>26.5 GPa</td>
<td>0.3 GPa</td>
</tr>
<tr>
<td>5. Width</td>
<td>76.29 mm</td>
<td>0.08 mm</td>
<td>11. Density</td>
<td>2800 kg/m³</td>
<td>30 kg/m³</td>
</tr>
<tr>
<td>6. D, 70S</td>
<td>17.21 mm</td>
<td>0.5 mm</td>
<td>12. Force</td>
<td>94490 N</td>
<td>500 N</td>
</tr>
</tbody>
</table>
FIGURE 4. Uncertainty results for recovering \( l\), \( m\) and \( n\) from three angle beam measurements as per the parameters of Table 3.

DISCUSSION

The forward problem time shifts shown in Figure 3(a) are in reasonable agreement with those calculated using literature values, and TOECs recovered using these time shift values are also in reasonable agreement. Note that there is considerably more scatter in the values for \( l\) in Figure 3(b) compared to \( m\) and \( n\) with the value of \( l\) from this work lower than the other two reported values. This is consistent with the sensitivity analysis showing that \( l\) has the largest uncertainty, which is in agreement with [10]. The sensitivity analysis shows that all of the resulting TOECs are very sensitive to errors in the L wave time shifts (2-3) and separation distances (7-8), with the uncertainty in \( l\) for parameters 2 and 7 (time shift and separation for the 45° L configuration) similar in magnitude to \( l\) itself. The analysis also confirms that only \( n\) has errors due to the SV time shift and separation distance because the two L-wave measurements are sufficient to perfectly recover \( l\) and \( m\).

The solution proposed here for the inverse problem is deterministic in that three measurements are required to determine the three constants, but there is considerable freedom in selecting the three measurement configurations. Four measurements (i.e., two shear and two longitudinal) lead to an over determined approach for recovering \( n\). The sensitivity analysis indicates that additional L measurements may be needed to reduce the uncertainty in \( l\), which would reduce the effect of measurement errors at the expense of experimental complexity.

SUMMARY AND CONCLUSIONS

A one sided liquid coupled method for determining third order elastic constants from a series of angle beam measurements has been proposed, the theory developed, and experiments performed which confirm the theory. This method offers several advantages over conventional acoustoelastic measurements such as one-sided access and easier transducer setup and alignment. A sensitivity analysis has shown that additional longitudinal measurements may be required to improve accuracy. Future work should concentrate on numerical investigation of the effect of different angle beam configurations on overall accuracy of recovered TOECs as well as measurements on additional materials of engineering interest for which TOECs are not known.
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REFERENCES