SIMULATION AND MEASUREMENT OF ULTRASONIC WAVES IN ELASTIC PLATES USING LASER VIBROMETRY

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ABSTRACT. The propagation of Lamb waves in elastic plates is analyzed both numerically and experimentally. A Scanning Laser Doppler Vibrometer (SLDV) is here used to detect and visualize transient waveforms propagating in an elastic plate at low ultrasonic frequencies. The waves are excited by a piezoelectric crystal glued to the plate surface and actuated by sinusoidal pulses of varying frequency. The pulse sequence is triggered by the SLDV internal controller so that phase and delay information are preserved. Such information allows visualization of the waveform pattern as it propagates over the plate surface. The experiment produces animated displacement maps where the interaction with discontinuities in the plate such as defects becomes apparent. This capability suggests application of the SLDV technique as part of an overall damage detection methodology which combines the recognized sensitivity of ultrasonic waves with the localization of damage via wavefield visualization. The interpretation of the experimental results is aided by numerical simulations of ultrasonic waves in plate structures. The simulations are performed using a Local Interaction Simulation Approach (LISA), which represents a simple and effective tool for simulating and visualizing waveforms in isotropic or orthotropic plate-like structures.

INTRODUCTION

Guided waves such as Lamb or Rayleigh waves have been proposed for damage detection purposes [1,2]. Their attractive features include good sensitivity to a variety of damage types and the capability of traveling relatively long distances within the structure under investigation. In previous investigations, guided waves are generated and received by transmit/receive transducer pairs distributed over the test specimen according to convenient patterns [3]. The fundamentals of this type of operation consist in evaluating the characteristics of the propagation along the wave path between each transducer pair. This technology has demonstrated its effectiveness in detecting small damage and discontinuities in the structure. The precise localization of damage however requires the application of additional algorithms which process the information obtained by a distributed array of sensors [3]. In the work presented here, the problem of detecting and simultaneously localizing damage is addressed by measuring Lamb waves with a Scanning Laser Doppler Vibrometer (SLDV) [4,5]. The SLDV system measures the out-of-plane velocity component of the scanned surface at points on a predefined grid. Scanning the grid and post-processing the data allows the detection and visualization of
the full wavefield as it propagates in the structure. The resulting images describe the main features of the propagating wave and show its interactions with discontinuities that may be encountered on the wave path. Moreover, the data required to generate the wavefield visualizations can be used to obtain time-of-flight information as well as to identify the structure and its propagation characteristics, such as for example wave and phase velocities and wave-number/frequency relationships. This paper presents work in progress which investigates the effectiveness and potential of the SLDV for wavefield detection. The experiments are performed on a thin aluminum plate for simplicity. The experimental investigations are supported by numerical simulations obtained through the Local Interaction Simulation Approach (LISA) [4,6,7]. LISA has been proposed as an alternative to Finite Element (FE) and Finite Difference (FD) methods for the efficient simulation of ultrasonic waves in homogeneous and heterogeneous solids [6]. LISA uses the same formalism as FD, but addresses the problem of discontinuities at interfaces by locally matching particle displacements and stresses, thus avoiding the difficulties of FD in dealing with large impedance mismatches at interfaces between different materials [6,7].

MODELING ULTRASONIC WAVE PROPAGATION IN THIN PLATES

Theoretical Background

The propagation of elastic waves in a homogeneous medium is governed by the well known elastodynamic wave equation, which can be expressed as [8,9]:

$$\mu \nabla^2 \mathbf{u} + (\lambda + \mu) \nabla \cdot \mathbf{u} = \rho \ddot{\mathbf{u}}$$  \hspace{1cm} (1)

where $\lambda, \mu$ are the Lamé constants, and $\mathbf{u} = [u_1 \ u_2 \ u_3]$ is the displacement vector. The assumption of a homogeneous medium is here made only for simplicity, and it is generally not required for the formulation of the simulation approach presented in the following section. The $p$-th scalar expression contained in equation (1) can be expressed in the following matrix form:

$$\rho \ddot{u}_p = \sum_{r=1}^{3} M_{pr} u_{p,rr} + N_{pr} u_{r,pr}$$  \hspace{1cm} (2)

where the notation $(.)_{p,rr}$ denotes partial derivatives with respect to the spatial coordinates $pr$, while dots indicate differentiation with respect to time. Also in equation (2), the matrices $M, N$ contain combinations of the Lamé constants, and their expressions can be found in [7-9].

The Local Interaction Simulation Approach

The Local Interaction Simulation Approach (LISA) proposed in [6,7] is here applied to simulate the propagation of elastic waves in the considered homogeneous medium. LISA utilizes the same formalism of Finite-Difference (FD) methods, but employs a different strategy which simplifies the problematic treatment of in-homogeneities and sharp discontinuities that often cause approximations and errors with FD methods. In LISA, the medium is first discretized into a lattice, for which elastodynamic equations are formulated. Discontinuities and in general changes to the material properties of the medium are treated by simply modifying the properties of the
lattice at the corresponding locations. The approach is very simple, easy to implement, and therefore very convenient for simulating wave propagation in heterogeneous media or in damaged specimens. Here, the LISA approach is used for simulating ultrasonic waves in thin aluminum plates. The goal of the simulations is to compare the obtained waveforms with those measured with the SLDV, and to obtain numerical results to be used for development and testing of damage detection algorithms.

In LISA, the propagation medium is discretized into a lattice with spatial step \( \Delta x = \Delta y = \Delta z = \varepsilon \), and in time with step \( \delta \). The discretized version of the spatial and time partial derivatives can respectively be expressed according to the well known FD formalism, which gives:

$$
\begin{align*}
\frac{u_{p,k}}{\varepsilon^2} = \frac{1}{\Delta x^2} \left[ u_{p}^{(k+1)} + u_{p}^{(k-1)} - 2u_{p}^{(k)} \right], \quad \frac{\ddot{u}_{p}}{\delta^2} = \frac{1}{\Delta t^2} \left[ u_{p}(t+1) + u_{p}(t-1) - 2u_{p}(t) \right].
\end{align*}
$$

Substituting equations (3) into equation (2) yields a set of three iterative equations which allow the computation of displacement \( u_{p} (p=1,2,3) \) at time \( t+1 \) for point \( i,j,k \), where \( t \) now is the discrete time index [7]. The displacements at time \( t+1 \) are a linear combination of the displacement components at \( i,j,k \) at time \( t \) and \( t-1 \), and of the point’s immediate neighbors at time \( t \). Hence, the computations for different points are completely independent of each other and the equations are suitable for parallel processing [7].

The LISA approach is here used to simulate the propagation of ultrasonic waves in a thin aluminum plate \((E=7.1\times10^{10} \text{ Pa}, \rho=2700 \text{ kg/m}^3)\). The considered structure measures 58 cm x 56.5 cm and is 1.5 mm thick as shown in Figure 1. Under the thin plate approximation, the assumption is made that all displacement components are constant through the thickness. This allows the out-of-plane displacement component to be decoupled from the in-plane behavior of the plate, which yields a significant reduction in the computational cost. This assumption is valid as long as frequencies belonging to the low ultrasonic range are considered, for which the associated wavelengths are large compared to the plate thickness. The considered plate geometry for obtaining numerical results replicates the experimental setup in which four piezoelectric discs bonded to the plate at selected locations can be used to generate ultrasonic pulses. The plate is discretized using a 400x400 lattice, and the piezoelectric discs are modeled as increased density and stiffness at the corresponding locations.

DETECTION OF ULTRASONIC WAVES USING LASER VIBROMETRY

The Scanning Laser Doppler Vibrometer (SLDV) is used to detect transient waveforms propagating at ultrasonic frequencies in the considered aluminum plate. The wavefield visualization and the large amount of available data have the potential to provide unprecedented information regarding propagation characteristics, such as time-of-flight, phase and group velocities, and details regarding the interaction of the wave with discontinuities caused by damage. The SLDV can therefore be used in conjunction with sparse arrays of ultrasonic transducers to provide the capabilities of detecting damage and immediately providing detailed information regarding its location within the structure.
Experimental Setup

The plate considered for the experiments is shown in Figure 1(a). Four piezoelectric discs are bonded to the plate and are used to excite elastic waves in the structure. The piezoelectric discs are driven by a function generator (Stanford Research Mod. SRS 360) through a voltage amplifier (Piezo Systems Mod. EPA-102). The excitation signals are 5-cycle sinusoidal bursts of varying frequency. The out-of-plane transient response is measured by the SLDV (Polytec PI, Mod. PV400 M2). For each scanning point a pulse is generated by the signal generator. The frequency of pulse generation is defined by a second function generator internal to the SLDV system. The latter is set to produce a low frequency signal (10 Hz) which triggers the pulse on the Stanford Research generator, and it serves as a reference signal for retaining the phase information required for post-processing the time signals. A schematic of the experimental setup is shown in Figure 2.

Experimental Results

The experimental setup described above is used to scan the plate and to detect its transient response. Examples of detected wavefields are shown in Figure 3. The responses shown in Figure 3(a) and 3(b) correspond to 5-cycle sinusoidal pulses at 50 and 150 kHz, respectively. The wavefields show the presence of one dominant mode of wave propagation, which corresponds to the first antisymmetric lamb wave mode \( (A_o) \) characteristic of this class of plate structures [9]. In the 150 kHz case, the second, faster mode corresponds to the first symmetric mode \( (S_o) \) of the plate. At this frequency the \( S_o \) mode is slower than at 50 kHz, and it can therefore be detected within the considered time window (0-0.4 ms).

FIGURE 1. Experimental setup (a) and schematic of plate and location of piezoelectric discs (b).
PRELIMINARY DAMAGE DETECTION INVESTIGATIONS

The numerical simulations obtained through the LISA approach can be directly related to the experimental results from the SLVD. The resulting wavefield information shows the influence of localized damage on the characteristics of wave propagation. The numerical simulations can be used for initial development of damage detection strategies and to validate data interpretation algorithms. The experimental measurements, which are usually affected by noise, demonstrate the effectiveness of the developed techniques. Initial attempts for data analysis and interpretation are here presented to lead the way to future, more extensive activities in signal and image processing.
**Numerical Results**

Wave propagation in a damaged plate is simulated by inserting a longitudinal slit in the lattice describing the elastic plate. The simulated slit is 25.4 mm long and measures 1.5 mm in width. It is modeled as a void in the lattice of the corresponding dimensions. The behavior of the wavefield corresponding to the out-of-plane displacements in the damaged plate can be observed from the snapshots shown in Figure 4. The distortions of the wavefield resulting from the reflections caused by the slit are effective indicators of damage, and clearly localize it on the plate surface.

A simple strategy for data post-processing consists in evaluating the Root Mean Square (RMS) value of the complete time history at each grid location and generating the corresponding map. The RMS values of the simulated out-of-plane response for the undamaged and damaged plate presented in Figure 5 show the effectiveness of this visualization and data-processing technique. In Figure 5(b), the presence and the extension of the slit are clearly visible. In both the damaged and undamaged plates, the distortions caused by the piezo discs are also very noticeable.

![Figure 4](image1.png)

**(a)** (b)

**FIGURE 4.** Snapshots of simulated wavefields in the damaged plate: (a) $t = 0.87$ ms, and (b), $t = 1.25$ ms.

![Figure 5](image2.png)

**(a)** (b)

**FIGURE 5.** RMS of simulated out-of-plane response: (a) undamaged plate and (b) damaged plate.
Experimental Results

Similar results are obtained experimentally using the SLDV. The wavefields of damaged and undamaged plates are measured, recorded and visualized for damage detection. Two types of damage are considered: two holes measuring 1/16” and 1/8” in diameter, and a 1/2” long slit. The slit is obtained by removing only part of the plate material thus avoiding cutting through the thickness.

Figure 6(a) and 6(b) show snapshots of the response of the plate with two holes, and with the holes and the slit. The time corresponding to the snapshot in Figure 6(a) has been selected so that the waveform corresponding to the applied pulse has already passed the location of the holes. It is clear that the two holes do not visibly affect the wavefield and therefore that more sophisticated signal processing tools are needed in order to improve the resolution of the technique. On the contrary, the presence of the slit is highlighted in Figure 6.b by both small amplitude reflections and distortion. Even in this case, however the detection of damage requires attention and may be obscured by higher noise levels in the data. The RMS values of the time-histories recorded at each grid location are computed and corresponding maps are generated. The RMS maps for the two types of damage considered are shown in Figure 7. Both maps clearly indicate the presence of discontinuities corresponding to the two types of damage considered. Of the two holes, however only the 1/8” hole is visible as a small bright spot, while the smaller hole still cannot be detected.

![Figure 6](image1.png)

**FIGURE 6.** Snapshots of experimentally measured wavefields: (a) plate with two holes and (b) with slit.

![Figure 7](image2.png)

**FIGURE 7.** RMS of experimentally measured response: (a) plate with two holes and (b) with slit.
CONCLUSIONS

The propagation of Lamb waves in elastic plates is analyzed both numerically and experimentally. Lamb waves have been demonstrated to be sensitive to discontinuities along their propagation path, and have therefore been proposed for damage detection purposes. This work combines numerical and experimental investigations on Lamb waves propagating in thin aluminum plates with known levels of damage. The numerical studies are performed using the Local Interaction Simulation Approach (LISA), which is a simple and effective tool for simulating and visualizing waveforms in isotropic or orthotropic plate-like structures. Simulated data are used for the generation of waveforms to be employed in the development and validation of damage detection algorithms. The experimental detection of propagating waveforms is performed by a Scanning Laser Doppler Vibrometer (SLDV), which is here applied to detect and visualize transient waveforms propagating in an elastic plate at low ultrasonic frequencies. The experiments produce animated displacement maps which can be directly compared with those obtained numerically with LISA. The wavefield data provide invaluable insight on the characteristics of wave propagation in the considered structure. In particular, the interaction of the propagating wave with defects becomes apparent. Preliminary results show how the resolution of the technique can be improved through the simple calculation of each signal’s RMS value. The resolution limits are here explored by considering two holes of increasing diameter and a slit partially cutting through the plate thickness. The presented results show the potential of the SLDV-based damage detection strategy. Future work will however need to address the evaluation of optimal pulse frequency, as well as the application of more refined signal processing and image recognition tools. The results presented in this work suggest the application of the SLDV technique as part of an overall damage detection methodology which combines the recognized sensitivity of ultrasonic waves with the localization of damage via wavefield visualization.

REFERENCES